

Comprehensive Overview and Comparative Assessment of Model-Based Methods for Vehicle State Estimation

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Article Info

Article history:

Received October 24, 2025

Revised November 6, 2025

Accepted November 17, 2025

Keywords:

Vehicle State Estimation,
Observer Design,
Parameter Estimation,
Sensor Fusion,
Model-Based Approaches

ABSTRACT

Accurate vehicle state estimation is essential for advanced driver-assistance systems (ADAS) and autonomous driving applications, enabling robust control performance under diverse operating conditions. In this context, it becomes even more critical to design generalizable tools capable of adapting to any vehicle, operating condition, or environment: for this reason, model-based approaches become a true necessity, especially when relying on the outputs of such algorithms in closed-loop control and safety applications, making these methodologies more suitable than purely data-driven approaches. However, in current literature, there exists a variety of model-based estimation strategies with different levels of complexity, accuracy, and computational requirements. This paper reviews approximately 200 studies and presents a comprehensive review of model-based vehicle state estimation techniques, with a focus on their formulation, underlying physical assumptions, and associated trade-offs. Estimation strategies are categorized into pure kinematic, vehicle- and tire-model-based, hybrid, and more complex multibody-model-based approaches, thereby facilitating informed selection of the most suitable method to support design choices based on sensor availability and application requirements. Within each category, a more detailed analysis examines estimation strategies with variable modeling assumptions, observer architectures, and sensor configurations, emphasizing how design choices affect accuracy, robustness to parameter variations, computational cost, and sensitivity to measurement inaccuracies. In contrast to previous surveys, this work provides a structured comparative assessment of all methodologies using specifically defined evaluation criteria, enabling a direct comparison across different estimation approaches and highlighting their respective advantages and limitations. The review concludes by outlining current limitations in the state of the art and identifying promising research directions, including the integration of currently unmodeled physical effects into estimation algorithms, the necessary steps toward simplified calibration and self-calibration procedures even in detailed modeling approaches, and the exploitation of emerging sensor technologies.

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1. Introduction

According to the World Health Organization (WHO), road traffic accidents remain one of the leading causes of death, with more than 1.3 million fatalities each year (Ahmed et al., 2023; Goel et al., 2024). In response to this pressing issue, the automotive industry is evolving toward increasing levels of vehicle automation, driven by the development of advanced driver-assistance systems (ADAS) and autonomous driving technologies (Channamallu et al., 2023; Nidamanuri et al., 2021). Research in this field aims not only to design efficient solutions that improve road safety but also to make driving automation accessible and integrated into everyday life, ensuring the mass production of these technologies (Indu & Aswatha Kumar, 2023; Teng et al., 2023).

In these applications, accurate knowledge of the vehicle's instantaneous state—including its kinematics, dynamics, and interaction with the environment—is essential to optimize the performance of control systems. In this context, virtual sensing plays a key role, enabling the estimation of quantities that are either not directly measurable or unavailable using traditional sensors typically equipped in mass-produced vehicles (Canale et al., 2008). As discussed in various studies examining future developments in this area (Guo et al., 2018; Jin et al., 2019), current research on vehicle state estimation focuses on developing algorithms that ensure stability and observability while maintaining performance under both linear and nonlinear conditions. A key objective is to balance algorithm performance with the consequent computational costs and sensor requirements, making these solutions suitable for cost-constrained and hardware-limited applications (Zarringhalam et al., 2013). Another significant aim is achieving robustness in the presence of unknown inputs and environmental conditions, such as friction variation and road profile changes (Grip et al., 2008; Na et al., 2017; Ray, 1997). In the current literature, various approaches are used for vehicle virtual sensing, each with distinct advantages and disadvantages. These methods are generally grouped into two broad categories: model-based and AI-based.

Model-based approaches rely on mathematical models of the vehicle and its environment, using physical laws to describe system behavior (Anderson & Bevly, 2010; Hashemi, Khosravani, et al., 2017; Jia et al., 2015; Reina et al., 2017; Törő et al., 2019). To ensure estimation stability, these models are typically embedded in state observer architectures—such as Kalman filters or Luenberger observers (Bishop & Welch, 2001; Zeitz, 1987)—that correct predictions based on real-time measurements, thus mitigating the accumulation of error in open-loop simulations (Bersani et al., 2019; Van Dong et al., 2019). While these techniques are generally accurate with well-calibrated parameters, they remain sensitive to environmental variations and uncertainties. Nevertheless, their predictable structure makes model-based methods well-suited for control system integration.

AI-based approaches, particularly those employing deep learning, are also increasingly employed for vehicle state estimation (Bonfitto et al., 2019; de Araujo et al., 2023; Feng et al., 2023; Jin et al., 2021; Kim et al., 2021; Kuutti et al., 2020; Pandharipande et al., 2023; Yu et al., 2022). These techniques can achieve high accuracy after extensive data collection and algorithm tuning. However, they typically require large training datasets and their lack of interpretability makes them challenging to debug and adapt. Moreover, AI-based estimators are often customized to specific datasets and vehicle setup, which limits their generalization.

Within these categories, each method presents specific strengths and weaknesses. The most appropriate choice of a technique for a given application depends on factors such as the sophistication of the available sensor technologies and experimental data for parametrization, and the architecture and requirements of the ADAS system in which the estimator is integrated. As a result, selecting an appropriate estimation strategy requires a careful balance of trade-offs across multiple performance dimensions, which will be discussed in more detail. In this regard, the current literature includes several reviews and comparative studies on vehicle state estimation, which play a crucial role in guiding the selection of appropriate estimators based on application constraints. The most comprehensive overviews available primarily focus on model-based approaches and are relatively dated, with the most recent one published in 2020. For instance, Singh et al. (Singh et al., 2019) provided a broad survey of vehicle state estimation methods, highlighting key modeling structures and observer design strategies. Jin et al. (Jin et al., 2019) and Guo et al. (Guo et al., 2018) extended the overview to include AI-based techniques, outlining general trends and future directions. However, these works lack direct performance comparisons across methods, and the evaluation of pros and cons mostly remains qualitative, limiting the usefulness for practical selection of estimation strategies. In contrast, more targeted comparative studies focus on the performance of specific estimators under defined conditions, often using experimental or simulated datasets as reference data for validation. These analyses typically concentrate on model-based techniques and compare the accuracy of estimators adopting different modeling formulations—such as kinematic versus dynamic models (Farrelly & Wellstead, 1996; Kang et al., 2014) or combined versus non-combined models (Cho et al., 2009). Other works examine the influence of observer design, comparing for example Kalman filters and Luenberger observers (Ndjeng et al., 2009; Park, 2024; Tao et al., 2021; Yang et al., 2017; Zhang et al., 2009) or study the effect of adaptive calibration strategies versus fixed-parameter filters, showing that adaptive schemes can enhance accuracy at the cost of higher computational complexity (Boada et al., 2019; Hong et al., 2014; Katriniok & Abel, 2015; Xiong et al., 2020; Zong et al., 2013). However, these comparative analyses remain limited to specific applications, typically focusing on a restricted set of state variables and lacking generalization across different estimation objectives.

Therefore, a comprehensive and comparative assessment of model-based estimation methodologies covering the entire state of the art is still absent in the literature. In particular, no previous review has systematically evaluated different estimator typologies using clearly defined performance criteria that can effectively guide the selection of the most appropriate strategy for a given application. This work aims to fill that gap by presenting a systematic review of model-based vehicle state estimation techniques, classified according to their modeling approach and degrees of freedom, and compared through a set of defined assessment criteria. These criteria have been designed to enable a

comprehensive comparative analysis between different estimation logics, considering different factors, including their accuracy in reproducing real vehicle dynamics, their dependency on parameter identification, their robustness with respect to external conditions such as sensor noise and road variability, and their computational complexity, which determines suitability for real-time implementation. The comparison has been conducted on multiple levels. At a higher level, the analysis identifies the advantages and limitations of each modeling macro-category, in which the existing literature has been organized according to the adopted modeling approach, providing guidance on which type of methodology best satisfies specific application constraints and performance requirements. At a lower level, for each estimation application and within each macro-category, the review compares individual estimation logics that differ in their modeling assumptions, observer formulations, and implementation details. AI-based approaches are explicitly excluded from this review, since their strong dependence on dataset quality and training strategies makes it difficult to establish objective and comparable evaluation criteria across different implementations. For this reason, the focus is restricted to model-based estimation techniques.

The paper is organized as follows:

- Section 2 describes the review methodology, including the search strategy, inclusion criteria, and selection procedure.
- Section 3 introduces the classification framework adopted for model-based estimators, describing the different levels of classification.
- Section 4 defines the performance indicators used to conduct the comparative assessment of the reviewed methodologies.
- Sections 5–8 provide a detailed and critical analysis of the methodologies included in each of the four modeling macro-categories (kinematic, vehicle- and tire-model-based, hybrid, and multibody observers). For each category, estimation methodologies are reviewed and grouped according to their application domain and estimation targets, and a comparative evaluation is performed using the uniform performance metrics previously defined.
- Finally, Section 9 summarizes the main findings of the review and outlines open challenges and future research directions.

2. Methodology

To perform a detailed classification and to cover the full domain of applications of interest, a systematic literature search was conducted using the main scientific publication databases, namely IEEE Xplore, Scopus, Web of Science, and Google Scholar, which ensure the availability of both journal articles and conference proceedings, and allow the identification of the most relevant contributions in terms of citation impact and recurring authors in the field.

Regarding the time frame, no strict temporal constraints were imposed. Foundational and general approaches were considered from works published before 2010-2012, while more detailed developments with additional features were included from 2010-2012 onward, to capture the latest advancements. Finally, to highlight emerging research trends and future perspectives, the search was restricted to the last five years.

The adopted keywords covered a set of combinations, including grouped terms with variable levels of generalization. General terms such as *vehicle dynamics*, *ADAS systems*, and *vehicle control* were used to frame the broader context. More specific keywords, directly related to virtual sensing, included *vehicle state estimators*, *vehicle virtual sensing*, *vehicle state observer*, and *vehicle model-based estimator*. Additional task-oriented keywords referred to the specific state variables under investigation, such as *sideslip angle estimation*, *slip ratio estimation*, and *grip estimation*, as well as to the modeling and observer approaches, including *single-track model* and *Kalman filter*. Furthermore, when the objective was to search methodologies with particular features, additional keywords were employed, such as *adaptive approach*, *hybrid estimator*, *kinematic approach*, *dual filter*, and *combined estimation*.

The selection process of the available articles followed this procedure. The multi-level classification, detailed in the following section, was established using the review papers reported in existing literature and further expanded to include emerging methodologies and technologies developed over the past five years that were not covered by previous surveys. For each estimation approach, categorized at the low level, the most cited and representative works were identified across the three previously defined time intervals. However, the inclusion of the most recent papers was restricted to those introducing clear differences in modeling formulations and sensors' setup compared to earlier studies, while works differing only in calibration procedures or very particular implementation aspects, such as the adoption of non-standard mathematical estimation methods, were not considered.

3. Model-Based Estimators: Classification Framework

Model-based estimation techniques, despite relying on different modeling assumptions and vehicle instrumentation, generally share a common workflow (Fig. 1). The process includes a prediction step, in which the

state is preliminarily updated through a physical model using the available inputs, followed by a correction step, where the prediction is updated based on the error with respect to instantaneous sensor measurements. Since the measured variables typically differ from the states of interest, measurement functions are required to relate estimated states to the directly measured quantities.

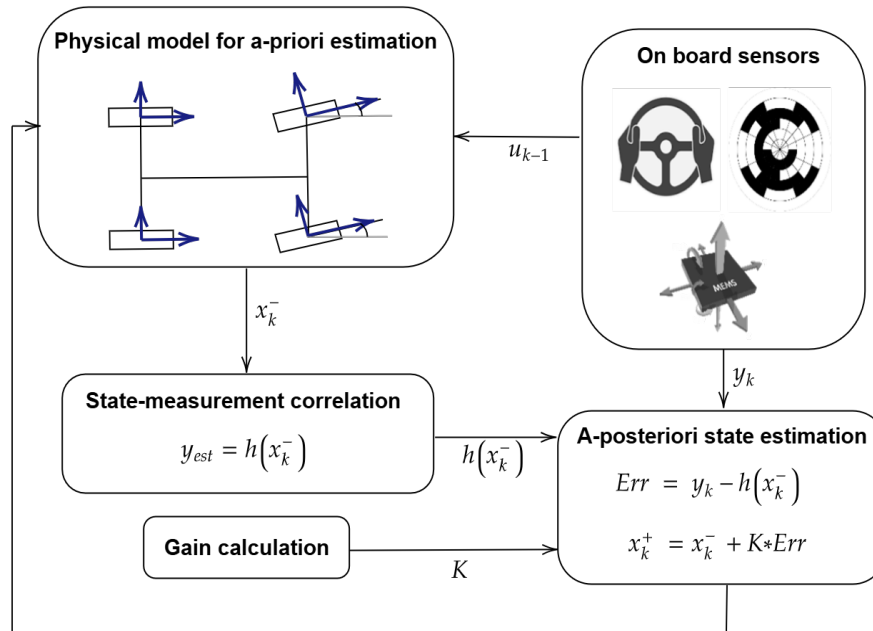


Figure 1: Schematic Representation of the General Model-Based Estimator Workflow.

The represented estimation workflow is a general reference, but it's not univocal, since the effective estimation strategy can present more complex workflows, resulting from the combination of multiple models or estimators and the integration of online signal processing techniques.

In this scenario, which includes a wide variety of approaches, a multi-dimensional taxonomy was adopted to enable the intended comparative analysis at multiple levels, which are graphically schematized in Fig. 2. The primary classification is based on four macro-categories of model-based estimators, reflecting the level of modeling detail and physical principles explicitly considered to perform the system prediction and correction:

- Kinematic-model-based: Utilizes kinematic relationships, employing acceleration and angular velocity signals to compute vehicle velocities.
- Vehicle and tire model-based: They can be categorized into tire-less and tire-model-based approaches; the first one relies on inverse vehicle dynamic equations without requiring tire modeling; the latter incorporates tire models to compute tire forces, which are then integrated into the vehicle equations of motion.
- Hybrid formulations: Combine kinematic and vehicle-model-based observers to optimize estimation performance by leveraging their respective strengths and mitigating weaknesses.
- Multibody-vehicle-model-based: Employs detailed multibody vehicle models explicitly accounting for suspension constraints and their influence on overall vehicle dynamics.

Beyond this preliminary grouping, the logics are distinguished between the main application domains of vehicle state estimation:

- Handling-oriented estimation: Focused on vehicle stability and maneuverability, typically addressing variables such as sideslip angle, yaw rate, or longitudinal slip. Within this domain, surface-related quantities can also be estimated, referring to tire-road grip condition (e.g., dry, wet, snow, ice).
- Ride and comfort-oriented estimation: Focused on out-of-plane vehicle dynamics and estimation of road unevenness. In the literature, these approaches are almost exclusively developed within the vehicle-model-based category.

A following classification level is then defined based on the specific estimation objective, consisting of the state variable or group of variables that are more relevant under particular operating conditions (e.g., straight-line or cornering maneuvers) or that are restricted to a specific set of vehicle degrees of freedom (e.g., longitudinal dynamics, lateral dynamics, combined dynamics, wheel dynamics). At this stage, another distinction is made between non-

adaptive observers, which operate with fixed model parameters, and adaptive observers, which are designed to identify varying parameters online. Finally, the lowest classification level concerns the individual modeling assumptions, sensor employment and implementation details.

4. Assessment Criteria

As previously declared, to pursue the aim of the review, comparisons across all the classified methodologies are made through specifically defined assessment criteria. A central aspect in this evaluation is the choice of the underlying model, which strongly influences estimator performance, but not in a straightforward manner, since it depends on the combination of several critical factors. On the one hand, simplified models with a limited number of parameters generally provide greater robustness to operating variability and are easier to generalize across different vehicle configurations. Their accuracy, however, is typically lower, and they rely more heavily on sensor quality, often requiring dedicated signal processing and careful gain tuning. Furthermore, reducing state variables means losing observation of local physical phenomena like individual wheel dynamics and kinematics, which are crucial in modern suspension- and tire-oriented control strategies.

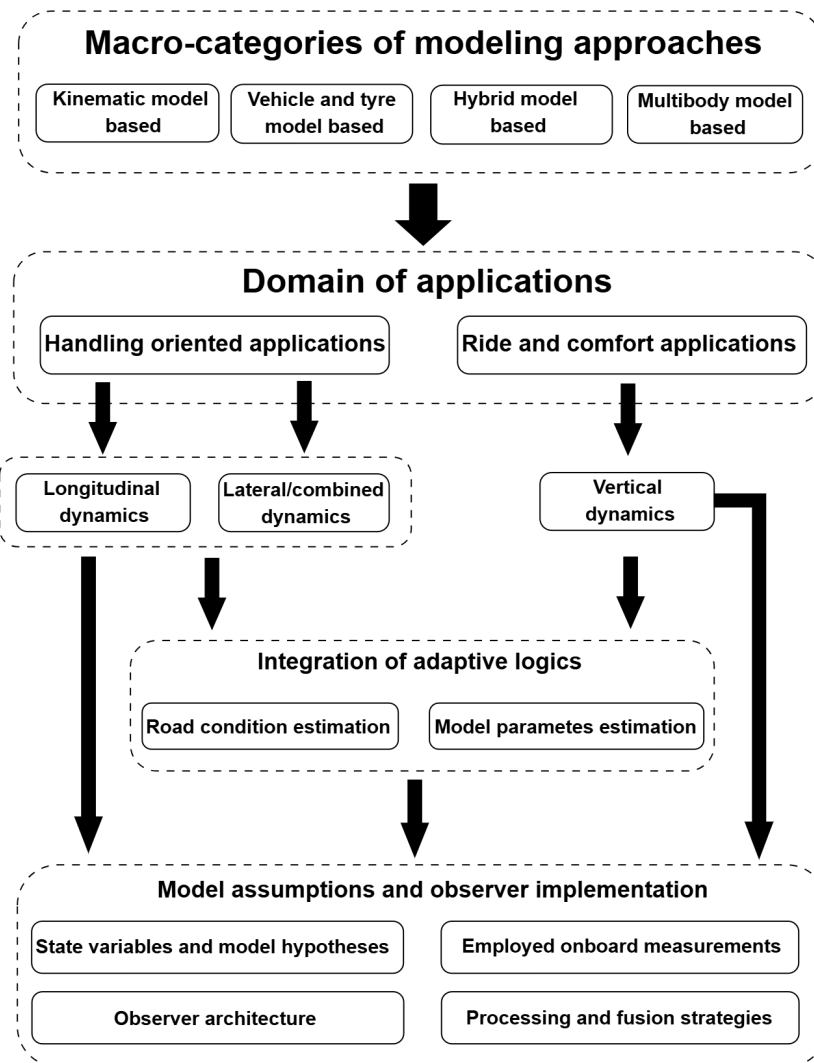


Figure 2: Classification Representation at Different Levels

On the other hand, more detailed models, explicitly accounting for tire dynamics, suspension effects, or nonlinear

interactions, can achieve higher estimation accuracy but at the cost of stronger dependence on parameter calibration and greater sensitivity to their variability. In these cases, adaptive algorithms are often required to preserve reliability under changing conditions. Moreover, highly nonlinear formulations demand complex observer structures, which increase computational cost and implementation complexity. Furthermore, increasing the level of details leads to the necessity of a higher amount of direct measurements to ensure observability.

Based on these considerations, the comparative analysis will be carried out according to the following performance metrics:

- *Modeling accuracy*: in the present review, accuracy is not assessed through direct comparison with reference data, since the metrics reported in the literature (e.g., RMSE, MAE), evaluated with respect to different datasets, compromise the objectivity of the comparison. Instead, accuracy is interpreted as the level of modeling detail adopted in the estimator formulation and on the physical aspects explicitly considered to predict its states. Six levels are defined:

- Level 1: only accounts for vehicle kinematics.
- Level 2: accounts for vehicle dynamics only.
- Level 3: accounts for both vehicle dynamics and tire dynamics.
- Level 4: accounts for vehicle dynamics and tire dynamics including nonlinearities and parameter dependencies on operating conditions.
- Level 5: models explicitly accounting for suspension effects under steady-state assumptions.
- Level 6: models explicitly accounting for suspension effects under transient dynamics.

For low-level incremental refinements of modeling detail, decimal values are added to the metric (e.g., Level 3.1, 3.2), providing a finer classification scale.

- *Robustness*: defined as the ability to preserve estimation performance under parameter variations related to changing road conditions and tire parameters. It can be classified as insensitive, sensitive or tunable through filter parameters.

- *Sensors' influence*: reflects the dependency of estimation accuracy on sensor errors (high, low, intermediate). It is defined by the sensors affecting the estimation and depends on the way they contribute to the state prediction or parameter estimation.

- *Computational cost*: it is defined by combining the degree of filter nonlinearity and the number of state variables. Four levels are introduced:

- Low: linear observers with a limited number of states (1-6).
- Medium: linear filters with a high number of states or nonlinear filters with a low number of states.
- High: nonlinear filters with a high number of states or highly nonlinear filters with a low number of states.
- Very high: highly nonlinear filters with a large number of states.

- *Sensor requirements*: defined as the amount and variety of sensors needed for the estimator to operate. Five levels are introduced:

- Level 1: in-plane IMU (longitudinal/lateral accelerations and yaw rate), wheel speed signals and steering angle.
- Level 2: in-plane and out-of-plane IMU channels (vertical acceleration, roll/pitch rates), wheel speed signals and steering angle.
- Level 3: Level 1 or 2 with the addition of one among GPS velocity, applied torque measurements, or suspension deflection sensors.
- Level 4: Level 1 or 2 with the addition of two of the above additional signals (GPS, torque, suspension).
- Level 5: full sensor setup including GPS, torque measurements, and suspension deflection, in addition to IMU, steering angle and wheel speeds.

- *Parametrization requirements*: defined as the number of calibration routine steps to guarantee consistent estimation. Five levels are introduced:

- Level 1: no calibration required.
- Level 2: identification of vehicle-level parameters (mass, inertia, wheel corner location and center of gravity position, global roll/pitch stiffness and damping).
- Level 3: identification of tire model parameters.
- Level 4: integration of suspension and steering kinematic and elastokinematic through static maps.

Level 5: full parametrization of suspension and steering systems, including geometry, inertia, and mechanical properties.

- *Operating domain*: defined as the range of operating conditions in which the estimator remains mathematically stable (typically proven through eigenvalue or Lyapunov stability analysis), observable (from rank

analysis), and reliable.

Once the criteria are defined, the comparative analysis is structured on two hierarchical levels to provide both a global and a local evaluation of estimator performance. At the higher level, the comparison is conducted among the main macro-categories of model-based estimators, to define general trends and identify the typical range of variation of the key performance indicators allowed by each modeling macro-methodology. This provides a first-level guideline on the advantages and limitations of each modeling approach and on their suitability depending on the application constraints. At the lower level, developed in the following sections, the comparison is then extended within each macro-category, analyzing and comparing the specific estimation logics. This second step aims to refine the analysis, highlighting the effects of particular modeling assumptions, sensor layout, observer structures, and additional features, such as online model parameter estimation or a combination of multiple modeling approaches, on the overall estimator performance.

Table 1: Estimator Macrocategories Comparison – Performance Evaluation

Observer	Modeling Accuracy	Robustness	Sensors' Influence	Computational Cost	Sensors' Requirements	Params Requirements
Kinematic	1	Insensitive	High to IMU and Wheel Speed Signals	Low	1–3	1
Tire and Vehicle Model Based	2–5	Sensitive–Tunable	Intermediate	Low–Very High	1–4	2–4
Hybrid	2–5	Tunable	Intermediate	Medium–Very High	1–4	2–4
Multibody	6	Sensitive–Tunable	Low	High–Very High	5	5

The first comparison, reported in Table 1, summarizes the results of the higher-level analysis, showing the range of performance indicators for each estimator category. The operating domain is not included, as it strongly depends on the specific estimation logic considered.

Kinematic estimators represent the most general-purpose solution, as they are largely independent of vehicle type and road conditions. However, their accuracy is inherently limited by the simplistic modeling assumptions and strong sensitivity to sensor noise and their low computational cost can also be achieved by some linear vehicle- and tire-model-based formulations, which already provide higher accuracy. For vehicle- and tire-model-based observers, parameter sensitivity is unavoidable in non-adaptive implementations, since parameters affect both state and measurement equations and their influence cannot be completely mitigated by the filter. Adaptive logics mitigate this issue, but robustness then depends on filter convergence time, which can be managed by the filter parametrization. Hybrid observers, combining a sensitive and an insensitive formulation, can tune their robustness via weighting logic. The computational burden of both the second and third categories, as well as sensor equipment and parameterization requirements, depends strongly on the complexity of the underlying vehicle/tire model. Compared to kinematic observers, they are less general, but the key performance indicators are variegated and can be managed by the implementation and modeling choices.

Multibody observers provide the highest modeling detail, but at the cost of maximum parameterization effort and extensive sensor setups, reflecting the larger number of degrees of freedom. Although emerging reduction and implementation strategies can alleviate computational cost, it remains higher than in the other categories. These approaches are therefore best suited when full vehicle knowledge is available and computational resources are sufficient.

Given this preliminary definition of the domain of suitability for each modeling approach, the following sections provide a systematic analysis within each macro-category, detailing the individual estimation logics and the respective trade-offs.

5. Kinematic-Based Approaches

Vehicle kinematic state observers focus on estimating the vehicle's longitudinal and lateral velocities (Kang et al., 2014; Ryu et al., 2013). Therefore, one of their primary limitations is the lack of information about the states of subcomponents, such as tire forces, suspension dynamics, and other internal physical quantities. These observers typically combine a model based on the kinematic relationships of rigid-body motion (as expressed by equation (1)),

implemented through numerical integration of acceleration signals and the gyroscopic terms associated with the angular velocities, with a virtual longitudinal velocity measurement. In the most general case, this velocity measurement is not directly available from a single sensor but is instead estimated by combining the individual wheel speed signals through sensor fusion techniques. However, some procedures in literature employ GPS signals (Bevly, 2004) to enhance accuracy.

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} + \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} a_x \\ a_y \end{pmatrix} \quad (1)$$

The primary strength of kinematic-based state estimators lies in their general-purpose formulation, applicable to all vehicle types and not requiring the identification of vehicle-specific parameters through experimental procedures. This characteristic makes them the most versatile methodology among those previously discussed, also considering their robustness to varying environmental conditions and vehicle operating scenarios. Moreover, their linear formulation enables implementation using simple estimation logics with low computational cost.

When only the longitudinal velocity is estimated, simpler techniques can be employed. Complementary filters leverage the frequency-dependent characteristics of sensors, combining time-integrated acceleration signals with wheel speed measurements (Panzani et al., 2012). The underlying assumption is that accelerometers provide more reliable data at high frequencies, while wheel speed sensors are more accurate at low frequencies. Thus, the acceleration signal is processed through a high-pass filter and the wheel speed signal through a low-pass filter, with the filtered signals then summed to produce the final longitudinal velocity estimate.

Direct fusion logics (Tanelli et al., 2006) adapt to the current driving scenario by dynamically selecting the most reliable input signals based on the estimated slip ratio of each wheel and the instantaneous vehicle conditions, without relying on observer architectures. When the vehicle is moving at low speed and the slip is negligible across all wheels, the longitudinal velocity is estimated by averaging the speeds of all four wheels. During acceleration, the estimator relies solely on the non-driven wheels, while, under braking conditions, all wheel speed data becomes affected by slip, and the estimator relies on acceleration integration.

When both longitudinal and lateral velocities are estimated, an observer structure becomes necessary. This involves an a priori velocity prediction based on kinematic equations, followed by an a posteriori correction step relying on a longitudinal velocity measurement obtained through fusion of wheel speed signals. Fusion methodologies include purely mathematical techniques based on fuzzy logic, which analyze deviations of instantaneous measured values from the average or previously estimated velocity (Savaresi & Tanelli, 2010). These methods also consider discrepancies between the time derivative of the wheel speed signal—obtained through a low-pass filter of the sensor output—and the vehicle acceleration signal to enhance accuracy (Daiss & Kiencke, 1995; Selmanaj et al., 2017b). Physical approaches instead select wheels based on the estimated maneuver: during acceleration, only non-driven wheels are considered, while during braking, wheels with the lowest slip ratio are preferred (Klomp et al., 2014; Oh & Choi, 2011; Tanelli et al., 2006).

Estimator gain calibration employs methodologies that optimize the mathematical robustness of the algorithm, as in pole placement estimation methods (Farrelly & Wellstead, 1996; Rezaeian et al., 2016; Selmanaj et al., 2017a), or by explicitly accounting for the sensors' accuracy, as in kinematic Kalman Filters. In pole placement methods, the gain is tuned by selecting an eigenvalue that ensures system stability, requiring experimental reference data to improve accuracy. In contrast, the Kalman Filter—relying on the Gaussian noise assumption—defines noise covariances associated with the model and measurements based on the employed sensors' characteristics.

However, the main limitation of the kinematic observer in estimating lateral velocity is its unobservability when the yaw rate approaches zero (Farrelly & Wellstead, 1996). This condition can be handled either by applying a heuristic method to identify rectilinear maneuvers based on yaw rate and steering input (Selmanaj et al., 2017a, 2017b) and forcing lateral velocity to zero, or by integrating the kinematic logic within a hybrid formulation that includes a dynamic estimator, as will be discussed in the following sections.

5.1 Acceleration Signals' Processing Techniques Accounting for Sensor Bias, Chassis Orientation and Road Gradient

The performance of kinematic estimators strongly depends on the reliability of the sensor signals. Sensor noise, bias, systematic errors, and instantaneous outliers can significantly degrade estimation accuracy. For this reason, several online signal processing techniques have been proposed in the literature and integrated into observer algorithms.

Accelerometers are typically affected by a bias that must be compensated online to prevent sensor drift (Batista et al., 2010). This can be achieved using a Kalman Filter based approach, where the bias is included as an additional

state variable (Panzani et al., 2012), or by analyzing the instantaneous discrepancy between the time derivative of the wheel speed signals and the measured longitudinal acceleration during straight line maneuvers. These procedures may either neglect or explicitly account for the gravitational effects caused by the vehicle body orientation and the road gradient (GROVES, 2008). In the former case, the estimated offset becomes time-varying, as it simultaneously includes the constant sensor bias and the variable contributions from the road profile and the body motions induced by the current driving maneuver. In the latter case, the estimation logic must be integrated with pitch and roll angle estimators, and, under the assumption of a non-flat road, with estimators of road slope and banking as well.

In (Selmanaj et al., 2017b), the pitch and roll angles are estimated using a steady-state model that requires knowledge of the pitch and roll stiffness, as well as the locations of the pitch and roll centers. The measured accelerations are used to compute the applied moments within the model equations. Since this methodology is valid only in steady-state, a low-pass filter is applied to the accelerometer. To account for high-frequency phenomena, a kinematic formulation is also employed and merged with the steady-state estimation with a complementary filter, employing angular velocity signals for numerical integration and high-pass filtering.

More detailed approaches involve implementing a state observer based on a dynamic model to estimate roll and pitch angles. This requires knowledge of pitch and roll damping, stiffness, and moments of inertia. The a posteriori correction is performed using angular velocity measurements, with the observer gain tuned through methods such as pole placement, sliding mode observers (SMO), or Kalman filtering (Kamal Mazhar et al., 2020; Rajamani, Piyabongkarn, et al., 2011; Zhao & Liu, 2014). While this model offers higher accuracy under both steady-state and transient conditions, it requires a more detailed characterization of vehicle parameters compared to the previous approach, which relies solely on pitch and roll stiffness and the location of the rotation axes.

Other methods, which are generally less accurate than the aforementioned approaches and rely solely on sensor measurements, do not require any knowledge of the vehicle's dynamic parameters (Eric Tseng et al., 2007; Oh & Choi, 2011; Xia et al., 2018). These methods perform an a priori estimation using a kinematic model, followed by an a posteriori correction based on reference pitch and roll angle estimates. For pitch estimation, the reference angle is obtained from the discrepancy between the longitudinal velocity update—derived from accelerometer signals—and the numerically filtered time derivatives of the wheel speed signals. For roll estimation, the correction is based on the difference between the measured lateral acceleration and the gyroscopic term associated with the yaw rate. However, the computed roll reference angle is reliable only under steady-state conditions. During transients, it is assumed that the kinematic formulation compensates for these limitations.

The same class of techniques is frequently employed to estimate road slope and banking angles by isolating the gravitational component from discrepancies in the acceleration signals (Ahn et al., 2024; Grip et al., 2009; Klomp et al., 2014). However, this task is complicated by the fact that the effects of road inclination are indistinguishable, from a sensor perspective, from those caused by vehicle pitch and roll. Therefore, to simultaneously observe body orientation and road gradient, the estimation framework must incorporate a dynamic model, capable of predicting the vehicle's body orientation based on acceleration measurements. This is typically implemented through Kalman Filters or unknown input observers (Hashemi, Zarringhalam, et al., 2017; Hyun & Cho, 2018; Ling et al., 2014). Similar to roll and pitch estimation on flat roads, pitch and roll rate signals are often employed through kinematic algorithms involving numerical integration and complementary filters, providing an additional orientation estimate that can be fused with the dynamic model-based estimation, using reference angles as measurement (Yang et al., 2020). In (Liao & Borrelli, 2019), the kinematic-based estimation of road bank angle is involved in an adaptive estimation strategy also including online estimation of accelerometer bias.

However, as it will be discussed in the next section, road gradient is estimated also in vehicle-model-based approaches, with more complete vehicle dynamic models to compute the total longitudinal and lateral forces acting on the vehicle—including tire and aerodynamic forces—and isolate the component associated with road inclination from the total measured acceleration (Hao et al., 2017; Ryu & Gerdes, 2004; Tseng, 2001). As already highlighted for approaches that incorporate dynamic tire models, these methods require a more detailed characterization of both vehicle and tire parameters.

5.2 Comparative Analysis

Table 2 summarizes the performance of kinematic-based observers. In this class of methods, the influence of sensors is always dominant: IMU signals are essential, especially for lateral velocity estimation, while wheel-speed processing is unavoidable unless GPS measurements are integrated. The latter provides an accurate reference for longitudinal velocity, thereby mitigating errors associated with wheel slip ratio and encoder noise.

For straight-line maneuvers, the simplest longitudinal formulations remain the most convenient choice, since the introduction of lateral dynamics leads to observability issues at zero yaw rate. However, when lateral velocity

estimation is required, kinematic observers become necessary.

The table shows how observers progressively account for more phenomena, starting from bias compensation, then including chassis motions (roll and pitch), and finally road gradient. Road gradient estimation, when performed without explicitly considering pitch and roll effects, is less accurate due to the coupling between chassis orientation and gravity.

Using basic sensors (in-plane IMU signals and wheel speeds), chassis orientation can be reconstructed by relying on steady-state dynamic models of pitch and roll. These methods require vehicle parameterization but do not need additional sensors; however, this is valid only in steady-state conditions. Accounting for transient phenomena requires the combination of the steady-state model with pitch and roll estimated kinematically by adding out-of-plane IMU signals.

On the other hand, if vehicle parameterization is not available, a sensor-based estimation can be performed combining a kinematic model with reference angles, but this configuration prevents simultaneous estimation of road gradient, which remains unobservable, and is very sensitive to signals' inaccuracy.

Finally, when both full IMU sensor setup and vehicle parameterization are available, the highest accuracy level can be reached, since road gradient and chassis orientation effects can be properly distinguished in both steady-state and transient conditions, combining dynamic model, kinematic equations and reference angles.

Table 2: Summary of Kinematic-Based Vehicle State Estimators – Performance Evaluation

Observer	Modeling Accuracy	Robustness	Sensors' Influence	Computation Cost	Sensors' Requirements	Params Requirements	Operating Domain
Pure Longitudinal Estimation	1	Insensitive	High to IMU and Wheel Speed Signals	Low	1	1	Straight-line Maneuvers
Pure Kinematic Observer (W/O Signals' Compensation)	1	Insensitive	High to IMU and Wheel Speed Signals	Low	1	1	Unobservable for Yaw Rate ≈ 0
GPS-Aided Pure Kinematic Observer	1	Insensitive	Intermediate to IMU	Low	3	1	Unobservable for Yaw Rate ≈ 0
Kinematic with Bias Compensation	1.1	Insensitive	High to IMU and Wheel Speed Signals	Low	1	1	Unobservable for Yaw Rate ≈ 0
Roll/Pitch Estimation Only Based on Sensors	1.2	Insensitive	High to IMU and Wheel Speed Signals	Low	2	1	Unobservable for Yaw Rate ≈ 0
Roll/Pitch Estimation Only Based on Steady-State Model	1.2	Insensitive	High to IMU and Wheel Speed Signals	Low	1	2	Unobservable for Yaw Rate ≈ 0
Roll/Pitch Estimation Fusing Kinematics and Dynamic Model	1.3	Insensitive	Intermediate to IMU and High to Wheel Speed Signals	Low	2	2	Unobservable for Yaw Rate ≈ 0
Only Road Gradient Estimation Using Reference Angles	1.2	Insensitive	High to IMU and High to Wheel Speed Signals	Low	1	1	Unobservable for Yaw Rate ≈ 0
Roll/Pitch and Road Gradient Estimation Fusing Kinematics and Dynamic Model	1.4	Insensitive	Intermediate to IMU and High to Wheel Speed Signals	Low	2	2	Unobservable for Yaw Rate ≈ 0

6. Vehicle and Tire-Model-Based Approaches

Vehicle and tire-model-based methods address key limitations of kinematic observers related to unobservability and sensor inaccuracies, especially during steady-state maneuvers and straight-line driving. These approaches integrate tire models to estimate longitudinal and lateral tire forces as functions of vehicle and wheel kinematic operating conditions and incorporate them into dynamic equations of motion to predict the time evolution of vehicle

state variables.

Estimating tire forces, compared to purely kinematic approaches, provides the additional advantage of enabling integration with diagnostic and adaptive algorithms aimed at assessing road surface conditions (e.g., available friction coefficient). Moreover, since these dynamic models are commonly employed in stability and traction control systems (Ahmed et al., 2021; Ahmed et al., 2020; Corno et al., 2015; Kwak et al., 2000) their incorporation into estimators facilitates integration with ADAS system architectures for vehicle handling and braking applications (Hsiao, 2012; Wang et al., 2018).

However, these methods are sensitive to errors resulting from parameter variability—particularly vehicle mass and tire model parameters—which are affected by factors such as wear, road surface conditions, thermal effects, and tire aging. For this reason, adaptive estimation algorithms are often employed to update such parameters online. However, these algorithms still require a reliable initial parameter set, typically obtained through offline identification from experimental data, to ensure fast convergence and robustness of the observer estimation. As previously mentioned, the multiple performance indicators of the observer are strictly related to the adopted modeling approaches.

From this perspective, the current section analyzes approaches with increasingly complex models, starting from the longitudinal and bicycle models, which provide a compact and linear representation of vehicle dynamics, facilitating real-time implementation and simultaneous estimation of macroscopic environmental variations such as road gradient and friction coefficient, but with limited observability of local physical phenomena like individual wheel dynamics or combined effects. More advanced double-track models progressively include additional effects which—although requiring more detailed knowledge of the system—allow monitoring of local quantities.

6.1 Model-Based Estimators of Longitudinal Dynamics

This class of estimators is mainly used in traction control and braking assistance systems, focusing on variables such as longitudinal velocity, wheel dynamics, slip ratio, and longitudinal forces. The adopted pure longitudinal formulations aim to perform the estimation of these state variables, avoiding the observability issues of combined and lateral models in straight-line maneuvers, especially when integrated with adaptive strategies of unknown parameters such as longitudinal friction coefficient, road slope, pitch angle, and vehicle mass. However, their applicability is limited to these conditions.

A key aspect is the estimation of the slip ratio. The simplest approach relies on a kinematic formulation, where longitudinal velocity is estimated using a kinematic method, and slip is computed from wheel angular speed and rolling radius. While this method is highly sensitive to sensor errors, it has the advantage of being usable in parallel with tire-less estimation of the longitudinal force acting on each wheel, when applied torques are known, thereby enabling slip-slope-based estimation of the maximum friction coefficient. In this approach—implemented in (Lee et al., 2004) using a Luenberger observer—longitudinal forces are modeled as random-walk variables and integrated into the wheel dynamics equations, and then corrected based on the error between estimated and measured wheel angular speeds. Alternatively, the forces can be estimated through a tire model using the kinematic slip ratio as an input, such as in the nonlinear observer used in (Zhao et al., 2017) and the maximum friction coefficient can eventually be estimated online. In (M'sirdi et al., 2006) a second order observer is used to estimate slip ratio, linear stiffness, and effective radius, when the tire works in the linear range.

The advantage of the described single-wheel observation procedures is that they do not rely on accelerometer measurements, thus avoiding the associated uncertainties. However, their accuracy depends on the reliability of an independently computed estimate of the longitudinal velocity, which benefits from the integration of GPS signals (Miller et al., 2001) which however is not always available.

To address this limitation, an alternative state update formulation is commonly adopted. It relies on a linearized expression of the slip ratio time derivative, which is independent of the vehicle's longitudinal velocity and used as the state variable (Fujii, 2006; Fujii & Fujimoto, 2007; Suzuki & Fujimoto, 2010), but depends on measurements of longitudinal acceleration, wheel angular velocity, and its time derivative. The slip ratio is then computed via numerical integration during the time-update step.

The a posteriori correction is based on the discrepancy between the measured acceleration—corrected for bias via an independent offset estimation—and the acceleration predicted by the tire force model. Therefore, besides assuming that the wheels are condensed into a single axle, this approach is robust when longitudinal forces act on a single axle both in braking and traction scenarios, to allow the information of the global vehicle longitudinal dynamics, ensuring the observability of the individual wheel dynamics. The degree of nonlinearity of the a-posteriori equations depends on the tire model. The implementation uses a Luenberger observer with a linear tire model in (Fujii & Fujimoto, 2007; Suzuki & Fujimoto, 2010; Vo-Duy & Ta, 2018), and a nonlinear Extended Kalman Filter (EKF) with the nonlinear Magic Formula tire model in (He et al., 2021).

In (Liang & Lin, 2012), the approach is made independent of the tire model, but correction is performed via additional state variables representing components of the slip time derivative directly derived from measured signals,

implemented in a Kalman Filter with process noise tuned according to the maneuver. In this last case, despite the advantage of not requiring tire model parametrization, the noise of the encoders becomes more influential because of the numerical derivative used to compute angular acceleration.

To overcome the hypothesis of forces acting on a single axle, (Y. Zhang et al., 2016) introduced a correction based on instantaneous torque and wheel velocity measurements, allowing the computation of forces in each wheel based on the wheel dynamic equation and without using a tire model. Then, the nonlinear tire model has been used to correct the slip ratio in the a posteriori step, employing the previously estimated forces as virtual measurements.

To also include longitudinal velocity in the estimation algorithm by making the system less sensitive to measurement error, dynamic longitudinal vehicle models are generally used, incorporating the calculation of longitudinal acceleration based on forces computed through tire models. Since in this case an accurate computation of slip ratio is required, knowledge or estimation of applied torques is required to perform a dynamic update of wheel angular velocity.

In (Rajamani, Phanomchoeng, et al., 2011; Rajamani, Piyabongkarn, et al., 2011), different state observer approaches have been used in this regard, involving variable measurements set up. When torque and GPS signals are both available, the time update of the wheel angular velocity is performed and the slip-based computed forces are used to update the vehicle longitudinal velocity by numerical integration. Therefore, GPS velocity and wheel speed measurements are used in the a posteriori correction. When torque and acceleration signals are used, the a posteriori correction is based on wheel speed measurements and on the error between the acceleration computed by slip-based forces, estimated through the tire model, and the acceleration signal. In this case, parallel longitudinal velocity and road gradient estimation are required. When GPS and accelerometer are used, both torques, with known repartition, and longitudinal forces are unknown input, and accelerometer, velocity and wheel speeds are used as measurements, ensuring observability. The advantage consists of the tire-less based approach, which does not require tire model knowledge, but a highly reliable velocity measurement is necessary, leading to the employment of GPS.

Another dynamic-based approach, completely independent of accelerometers (Tanelli et al., 2012) used the tire-based computed longitudinal acceleration to perform the best wheel selection for estimating vehicle velocity from wheel speed measurements. Especially in this last case, high-fidelity tire modeling is required.

To further enhance accuracy, more detailed modeling approaches are available, accounting for tire transient dynamics and low-frequency suspension influences. A dynamic model-based approach for estimating longitudinal velocity is presented in (Rajendran et al., 2019), where wheel, simplified suspension, and vehicle dynamics are jointly modeled. The method uses a high-order sliding mode observer (HSMO) to estimate vehicle velocity and external disturbances, and an EKF to estimate vertical tire forces from a vertical dynamics model that accounts for suspension effects, road slope, and pitch under severe braking. The effective tire radius is adaptively updated as a function of vertical load, improving slip estimation accuracy. While this approach enhances the accuracy compared to the state of the art, it is less generalizable since it requires accurate knowledge of vehicle subcomponents and radius dependencies, also increasing the complexity and nonlinearity of the filter. However, it requires a sensor setup commonly available. In (Do et al., 2024), the tire dynamic-based approach for estimating longitudinal velocity is enriched by introducing the relaxation length concept to estimate the actual friction coefficient and longitudinal velocity, explicitly accounting for transient dynamics, using torque measurement as input and acceleration and wheel speed measurements as feedback.

6.1.1 Model-Based Estimators of Longitudinal Dynamics: Adaptive Logics for Mass and Slope Estimation

As previously discussed, longitudinal model-based observers can be extended to identify key vehicle parameters—most notably mass and road slope—which directly affect the longitudinal force balance. For this reason, adaptive logics have been introduced to estimate them online, exploiting the same model–measurement discrepancies used for kinematic state estimation.

Mass estimation is typically based on the rearrangement of the longitudinal dynamics equation, where the observer state vector is augmented by including vehicle mass as a time-varying parameter. Estimation relies on the minimization of the discrepancy between the measured longitudinal acceleration and the acceleration predicted by the vehicle's dynamic model—this model includes traction forces computed through a tire dynamic model, resistive forces (such as aerodynamic drag and rolling resistance), and the gravitational component if slope is known or negligible.

Adaptive estimation methods often employ recursive least squares (RLS) (Fathy et al., 2008; Jensen et al., 2022; Lin et al., 2018) typically relying on torque and wheel speed sensors to compute tire forces, and the state correction relies on acceleration signals and optionally GPS velocity for enhancing accuracy (Bae et al., 2001). However, the mass estimation is influenced by the road slope because the accelerometer measurement is affected by it, and because the slope directly affects the longitudinal force balance. Therefore, the slope can be estimated independently by exploiting the discrepancy between the measured longitudinal acceleration and the derivative of the longitudinal

velocity (Lingman & Schmidbauer, 2002). Alternatively, when GPS is not available and acceleration measurements are highly inaccurate, mass and slope can be simultaneously estimated using RLS with multiple forgetting factors (Vahidi et al., 2003; Vahidi et al., 2005) using torque and wheel speed measurements.

6.1.2 Model-Based Estimators of Longitudinal Dynamics: Comparative Analysis

Table 3 summarizes the comparative evaluation of longitudinal dynamic-based estimators. The first two rows, which only differ in sensor setup (with or without GPS aiding), represent the most general solutions, particularly suitable for estimating longitudinal grip. Their main limitation lies in the influence of sensor equipment, where GPS implementation is necessary to reduce the sensitivity to encoders noise, otherwise influencing simultaneously longitudinal speed and wheel angular velocity in the slip ratio formulation.

For applications demanding higher accuracy, nonlinear tire-model-based approaches become necessary. These, however, require torque measurements, since relying only on wheel encoders significantly compromises accuracy.

Slip-derivative-based formulations represent an intermediate solution: they reduce sensitivity to wheel-speed errors since they enable insensitive formulations with respect to longitudinal velocity. Nevertheless, these logics are constrained to a single-axle force, unless torque measurements are required. In both cases, balancing between tire-less and tire-model-based formulation depends on observer parameters, which can therefore leverage the influence of parameters and sensors’ uncertainty.

Further accuracy can be achieved when the longitudinal velocity is explicitly included in the estimation, implementing a full vehicle longitudinal dynamic model, but this increases computational cost and generally requires GPS support if the tire model is unknown. Additional accuracy is obtained when the estimator adapts vehicle mass; however, this strategy demands very accurate knowledge of both the tire model and simultaneous estimation of the road slope to avoid systematic errors. Furthermore, it necessarily implies either a complete sensor setup to ensure observability or the adoption of more complex and computationally demanding algorithms based on multiple forgetting factors.

6.2 Estimators of Lateral and Combined Dynamics

6.2.1 Estimators of Lateral and Combined Dynamics: Tire-Model-Based Approaches

The simplest tire-model-based formulation for lateral dynamics employs a 2-degree-of-freedom (2-DOF) single-track model (Guiggiani, 2014), where the sideslip angle and yaw rate are defined as state variables, while the steering angle and longitudinal velocity—estimated separately through the previously mentioned approaches—are used as inputs. When a linear tire model is adopted, the resulting system remains linear and can be incorporated into straightforward observer designs, such as Kalman Filters or Luenberger observers (Stephant et al., 2004; Venhovens & Naab, 1999). Additionally, alternative observer architectures have been proposed in the literature, in which the observer gain is explicitly defined to reduce sensitivity to variations in front or rear cornering stiffness (Farrelly & Wellstead, 1996).

Table 3(a): Summary of Longitudinal Dynamic-Based Vehicle State Estimators – Performance Evaluation

Observer	Model acc.	Robustness	Sensors’ Influence	Comp. Cost	Sens. Req.	Param. Req.	Operating Domain
Kinematic Slip Estimation with Tire-Less Model Force Estimation Based on Wheel Dynamics	3	Insensitive	High to Wheel Speed Signals	Low	3	2	Straight-Line Maneuvers
Kinematic and GPS-Aided Slip Estimation with Tireless Model Force Estimation Based on Wheel Dynamics	3	Insensitive	Intermediate to Wheel Speed Signals	Low	4	2	Straight-Line Maneuvers
Kinematic Slip Estimation with Tire-Model-Based Force Estimation	4	Sensitive	High to Wheel Speed Signals	Low	3	3	Straight-Line Maneuvers
Slip Derivative-Based W/O Torque Measurements	4.1	Tunable	Intermediate To Wheel Speed and IMU Signals	Low	2	3	Straight-Line Maneuvers (Forces on Single Axle)

Table 3(b): Summary of Longitudinal Dynamic-Based Vehicle State Estimators – Performance Evaluation

Observer	Model acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.	Operating Domain
Slip Derivative-Based with Torque Measurements	4.2	Tunable	Intermediate to Wheel Speed Signals	Low	3	3	Straight-Line Maneuvers
Full Longitudinal Dynamic Model with GPS and Torque	4.4	Sensitive	Low	Medium	4	3	Straight-Line Maneuvers
Full Longitudinal Dynamic Model with Accelerometer and Torque	4.4	Sensitive	Intermediate to Wheel Speed Signals	Medium	3	3	Straight-Line Maneuvers
Full Longitudinal Dynamic Model with Accelerometer and GPS	4.3	Insensitive	Intermediate to IMU	Medium	3	2	Straight-Line Maneuvers
Full Longitudinal Dynamic Model with Higher Modeling Details	4.5–5	Sensitive	Intermediate to Wheel Speed Signals	High	2–3	3–4	Straight-Line Maneuvers
Mass and Slope Estimation with Accelerometer and W/O GPS	4.6	Sensitive	High to Wheel Speed and IMU Signals	Medium	3	3	Straight-Line Maneuvers
Mass and Slope Estimation with Accelerometer and GPS	4.7	Sensitive	Intermediate To Wheel Speed and High to IMU	Medium	4	3	Straight-Line Maneuvers
Mass + Slope RLS Multi-Forgetting Factors	4.5	Sensitive	High to Wheel Speed Signals	High	3	3	Straight-Line Maneuvers

Observability of the algorithm is typically ensured by incorporating both yaw rate and lateral acceleration measurements into the a posteriori correction step. However, as shown in (Stephant et al., 2004), configurations relying on only one of these two measurements are also feasible, with observability demonstrated within specific ranges of vehicle parameters.

The longitudinal velocity can also be included among the state variables and estimated using different strategies. The most generalized approach employs a kinematic update, where the longitudinal acceleration measured by the accelerometer is used as input for numerical integration. In these cases, the acceleration signal can also be used to estimate axle forces, supposed as random-walk variables (Prakash & Dheer, 2023). Alternatively, a longitudinal dynamic model can be implemented within the formulation which however is influenced by errors in slip ratio computation.

If a nonlinear tire model is employed, the structure of the estimator can still rely on the same set of state variables, measurements, and inputs. However, the nonlinear nature of the model necessitates the use of nonlinear filtering techniques. In (Gadola et al., 2014; Huang et al., 2017; Liu & Cui, 2022) a single-track vehicle model with Magic Formula tire model is integrated into an Extended Kalman Filter (EKF) framework to estimate sideslip angle and yaw rate. Further implementation strategies include a Dugoff tire model adopted within a Scaled Kalman Filter with Model Error Compensator (SKFMEC) (Han & Huh, 2011), a Burckhardt tire model in a nonlinear observer (Baffet et al., 2007) and a UniTire model within an UKF (Zhu & Zheng, 2008).

Although nonlinear tire modeling provides improved accuracy and applicability in a wider operating domain, the mentioned implementations are computationally more demanding and the models are more sensitive to parameter uncertainties, particularly because variations in lateral grip predominantly affect the nonlinear region of tire behavior. Furthermore, they still provide reliable estimation only during steady-state conditions. To improve estimation accuracy under transient conditions, some studies incorporate the concept of tire relaxation length in the modeling of tire sideslip angles. In such cases, nonlinear filtering techniques become necessary even when using a linear tire model (Kim, 2009; Kim, 2010).

Despite the robustness of the single-track model in estimating vehicle kinematics at the center of gravity, the adoption of a double-track model (Guiggiani, 2014) becomes essential when estimating forces at individual tires and when accurately capturing nonlinear effects such as lateral load transfer, which are particularly significant when using

nonlinear tire models. In this formulation, observability remains ensured when the lateral dynamics are excited ($\delta \neq 0$ and $u \neq 0$); however, the overall system becomes more complex and several additional aspects have to be taken into account.

First of all, in single-track formulations, where the estimation of corner-level forces is not required, the steering system is typically modeled by applying a constant steering ratio that maps the steering wheel angle to the equivalent front axle angle (Guiggiani, 2014); alternatively in some works, the steering angle is not used as input but is instead estimated by integrating the steering system modeling through observers and data-driven transfer functions (Krishnaswami & Rizzoni, 1995; Ma et al., 2018; B. Zhang et al., 2016). As a result, in both cases, the tire model does not represent the behavior of the individual tire but rather a simplified effective axle characteristic. Instead, in double-track models, the accuracy of tire force estimation strongly depends on how the individual wheel steering angles are determined. Simplified formulations often rely on a constant steering ratio at each wheel (Acarman, 2008; Reif et al., 2008) introducing an approximation similar to single-track models and requiring the use of an effective axle-level tire model instead of representing individual tire behavior.

More accurate approaches incorporate explicit steering and suspension system models by using static maps derived from the known geometry (Antonov et al., 2011). Although these methodologies significantly enhance accuracy in individual force calculation, they require a more detailed knowledge of the vehicle characteristics. Moreover, their reliability, confined in steady state conditions, represents a key limitation of double-track approaches when compared to multibody formulations, which are capable of modeling suspension influences dynamically and with higher fidelity under transient conditions.

Another crucial aspect concerns the estimation of longitudinal forces at each wheel, which becomes necessary in double-track modeling, further complicating the resolution of the lateral and yaw dynamics estimation problem. The methods available for this purpose exhibit the same strengths and limitations previously discussed in the context of pure longitudinal dynamics modeling.

In (Doumiati et al., 2009a, 2009b; Doumiati, Victorino, Lechner, et al., 2010; Doumiati, Victorino, Charara, et al., 2010; Wang et al., 2012), a Dugoff tire model is employed to estimate the lateral tire dynamics, while the longitudinal forces are inferred from the measured longitudinal acceleration and the known distribution of traction or braking torque between the front and rear axles. Both EKF, UKF and Particle Filters have been employed. The main advantage of this approach lies in its independence from the numerous uncertainties associated with longitudinal tire force modeling. However, since individual tire forces are not observable, the estimation is performed at the axle level, and the influence of yaw rate on longitudinal force distribution is neglected.

Conversely, in (Dakhllallah et al., 2008; Li & Zhang, 2016; Zhao et al., 2010), the longitudinal Dugoff model is explicitly integrated into the estimation framework, using wheel angular velocity measurements to compute slip ratios. On the one hand, this approach introduces sensitivity to inaccuracies in wheel speed measurements, which can increase estimation uncertainty. On the other hand, if the primary objective is to maximize the accuracy of velocity estimation rather than that of tire force estimation, the feedback provided by the measured acceleration and longitudinal velocity helps maintain the estimation stability and robustness. Furthermore, implementations based on more advanced filtering techniques can further improve accuracy, even when using more complex and nonlinear tire models. For example, in (Liu et al., 2016; Liu et al., 2021; Ren et al., 2014; Zhang et al., 2019; Zhang et al., 2022), UKF-based and RLS-MHE approaches integrate longitudinal dynamics with Magic Formula tire models to estimate tire forces, relying on wheel speed measurements as inputs, while in (Wang et al., 2021) an adaptive fault tolerance EKF is implemented with the same modeling approach.

To enhance tire longitudinal dynamics calculation, wheel dynamic equations are integrated, involving the two main approaches already discussed in pure longitudinal estimators. The first one is the tire-less model based on wheel dynamics equations, employing torque and wheel speed measurements to compute longitudinal forces; the second one includes the slip-based calculation of longitudinal forces, but with wheel angular velocity estimated through wheel dynamic equations relying on torque signals in input and tire-model-based force computation. The former remains preferable when robustness to estimation error of longitudinal velocity and tire model parameter uncertainties is prioritized. In (Xu et al., 2025) a sliding mode observer is implemented for computing the longitudinal force in each wheel and provided as input to an EKF filter estimating velocities and yaw rate.

On the other hand, the second approach enables the accurate estimation of slip ratios, which are essential for implementing combined-slip tire models, enhancing the accuracy in the estimation of lateral dynamics, and in the estimation of longitudinal friction coefficient for road classification. In this regard, in (Heidfeld et al., 2020) and (Antonov et al., 2011) UKF estimators are used to simultaneously estimate forces, wheel and vehicle dynamics and road friction coefficients, using a nonlinear combined tire model. In (Xue et al., 2022), the same model-based approach is used in a RM-UKF to estimate wheel and vehicle state variables, employing a combined Magic Formula

to estimate forces.

When torque measurements are unavailable, they can be treated as unknown inputs, as shown in (Ouahi et al., 2013) and (Moaveni et al., 2015), where nonlinear observers with proven convergence are employed.

6.2.2 Estimators of Lateral and Combined Dynamics: Tire-Less Approaches

Alternative methodologies based on single-track modeling assumptions employ tire-less model estimators, where lateral axle forces are directly inferred from lateral acceleration and yaw rate signals. In (Baffet et al., 2008) and (Cho et al., 2009) these forces are modeled as random-walk variables and corrected using a Sliding Mode Observer (SMO) and a Kalman Filter, respectively.

This approach allows for the design of estimators that are robust to variations or uncertainties in tire model parameters, and it remains effective across both linear and nonlinear operating regimes. It is often combined with tire-model based methods to enhance parameter estimation accuracy.

However, a significant limitation lies in the lack of observability of individual wheel lateral forces, which requires the adoption of approximate assumptions to distribute axle forces among the wheels. This distribution is typically based on vertical load ratios (Wilkin et al., 2006); however, such methods, in addition to neglecting differences in left and right tire sideslip angles, become inaccurate during aggressive maneuvers where load dependency exhibits strong nonlinearity. To address the limitations of vertical-load-based force distribution, (Rezaeian et al., 2014) proposes a more complex cascade observer: following the initial axle force estimation via the bicycle model, an Extended Kalman Filter (EKF) is employed to estimate the random-walk error between the distributed forces and the actual wheel forces. Alternatively, a simpler heuristic approach is proposed in (Yang et al., 2014) where adaptive correction factors are introduced to compensate for the nonlinear load dependency, effectively improving the distribution accuracy without relying on a full tire model.

To simultaneously estimate wheel longitudinal forces, analogous tire-less procedures are integrated. These are based on modeling longitudinal forces as random-walk variables and utilize torque and wheel speed measurements (Albinsson et al., 2014; Cho et al., 2009). Such logics are of critical importance in ESP applications involving trajectory management through asymmetric longitudinal forces applied to the tires.

6.2.3 Estimators of Lateral and Combined Dynamics: Adaptive Strategies

As mentioned earlier, online adaptation of tire model parameters, such as stiffness and maximum available grip, is essential both when offline calibration is not available and a self-calibration procedure is further necessary, even when a reliable starting set is known due to their variability with temperature, wear, and asphalt conditions.

A widely adopted strategy involves the combined use of tire-less and tire-model-based estimators. The former provides a preliminary estimation of tire forces, which are then used as additional measurements in the tire-model-based estimators. In these cases, the latter usually rely on simplified vehicle models and filter architectures.

When the vehicle operates within the linear range of tire dynamics, a common approach is to use a single-track tire-less model to estimate lateral axle forces. Given a linear tire model, these forces enable the reconstruction of axle sideslip angles, which can then be translated into an estimate of the overall vehicle sideslip angle using steering input, longitudinal velocity, and yaw rate. In (Oh & Choi, 2011; You et al., 2009) the authors use instantaneous discrepancies between the sideslip angles computed at the front and rear axles to perform online adaptation of the cornering stiffness at each axle. The adaptation is governed by stability-proven adaptive laws, but the method is limited to the linear operating region, where the underlying model assumptions hold.

In (Baffet et al., 2008) a different approach is proposed, based on a tire-less observer using a Sliding Mode Observer (SMO) to estimate lateral forces. These forces are then employed as pseudo-measurements in a cascade structure, which simultaneously estimates the vehicle sideslip angle and the deviation of axle cornering stiffness from their nominal values. The EKF enables real-time stiffness adaptation even under nonlinear conditions, but—unlike nonlinear models, which define fixed force-slip relationships—it requires a convergence interval even under constant road conditions when tire kinematics move in a nonlinear region.

When the aim is to estimate fixed parameters in the nonlinear tire operating range, the problem is typically addressed by estimating the maximum available grip using a nonlinear tire model, such as the Dugoff model, in simultaneous estimation with sideslip angle and yaw rate, as in (Ding et al., 2014), which is based on a nonlinear Luenberger observer. However, the adaptive strategy is interrupted when the model operates in the linear range, where grip is not observable.

With the described fusion methodologies, simultaneous estimation of longitudinal tire model parameters can be performed, even in combined maneuvers. An example is found in (Chen et al., 2022) where the forces acting at each

corner are estimated through torque and wheel speed measurements and used to adapt a nonlinear longitudinal tire model.

The reported cascade estimation strategies rely on fully known mass and inertia parameters—which are essential to accurately estimate forces from acceleration measurements—and on the separate estimation of roll and road banking to correct acceleration signals, as is also done in kinematic estimators. Similarly to the latter, the underlying principle is to identify systematic discrepancies between predicted and measured accelerations, and to integrate a pitch/roll model to distinguish between the effects of chassis motion and those due to road gradient (Hyun & Cho, 2018).

On the other hand, mass and inertia can also be estimated online based on discrepancies on lateral acceleration and yaw rate prediction. However, as with mass estimation based on longitudinal dynamics, the accuracy depends on the knowledge of tire model parameters and road gradient, which must be either fully known or simultaneously estimated. The last requirement entails the use of more complex filters, organized into different architectures depending on the estimation goals, used to simultaneously estimate multiple parameters.

Multiple model approaches rely on a set of parallel observers, each associated with a specific parameter set (e.g., combinations of grip and cornering stiffness). A model selection or fusion mechanism—often based on likelihood evaluation or residual minimization—identifies the model most consistent with the observed dynamics (Athans & Chang, 1976). The stability of the algorithm benefits from the implicitly bounded parameter variation domain. However, the computational cost increases with the number of models. Additionally, convergence is guaranteed only for one of the employed parameter sets, which is generally not the optimal one. As a result, these methods are well-suited for adapting to changes in discrete road conditions but are not ideal for tracking intermediate variations in tire parameters due to temperature or wear. Moreover, for tasks such as auto-tuning of constant parameters (e.g., mass, inertia, or nominal stiffness) or accurate instantaneous estimation of variable quantities (e.g., roll and bank), alternative approaches are typically preferred. Consequently, the primary applications of these strategies, whose implementations are usually based on nonlinear Kalman Filters, involve the identification of discrete grip and stiffness levels, with the different models corresponding to distinct classified road conditions (Tsunashima et al., 2006; Tufano et al., 2024).

Dual filter architectures decouple state and parameter estimation: one filter estimates vehicle states using previously estimated parameters, while a secondary filter updates parameters based on the innovations or residuals. This separation helps maintain estimation stability and enables gradual parameter adaptation, particularly effective under slowly varying conditions. Therefore, they are suitable for online auto-calibration of the tire parameters, when fully unknown and supposed to be constant, such as in (Lee et al., 2015) and (Zong et al., 2013), where tire stiffness and grip are estimated respectively and in (Davoodabadi et al., 2014), where the MF-model parameters are simultaneously identified. Furthermore, because of their efficiency in estimating constant parameters, they are frequently used for combined estimation of lateral dynamics and mass and inertia for online identification of inertia vehicle properties (Jin et al., 2023). However, the convergence interval depends on initial conditions and filter tuning, as the adaptation rate is influenced by the noise covariance matrices, which ideally should be adjusted in tandem with the parameter update process. Therefore, they are not robust when quick online parameter adjustments due to road conditions' variations are required.

In contrast, joint observer methods augment the state vector to include unknown parameters and estimate them together with dynamic states in a single filter. This coupling enables more responsive adaptation, then suitable for estimating quickly variable quantities, but the larger number of state variables can lead to unobservability. Therefore, they are frequently employed with single track vehicle model to reduce the observed quantities. This kind of architecture is employed in (Reina & Messina, 2019) to estimate tire lateral stiffness in both front and rear axles, while in (Berntorp et al., 2021) maximum friction coefficient and vehicle mass are estimated with proven maintained observability.

6.2.4 Estimators of Lateral and Combined Dynamics: Comparative Analysis

Table 4 reports the comparison among the different approaches for lateral and combined dynamics estimation. The single-track model represents the simplest solution from the computational point of view and the most effective when compatibility with control logics must be maximized. Its integration in parallel with tire-less logics based on the bicycle model, or within adaptive formulations in both the linear and nonlinear range, provides the best trade-off when the objective is to maximize the speed of parameter estimation and thus road condition detection, while avoiding observability issues and limiting computational burden.

Conversely, the double-track model is employed when accuracy and the observation of local phenomena are prioritized. From the table, which reports logics with increasing accuracy, two crucial aspects emerge that characterize this approach:

- Longitudinal force estimation: introducing slip-dependence increases sensitivity to errors. If the application

does not require accurate force estimation (for example, when the main objective is the estimation of longitudinal velocity), this limitation is acceptable. Otherwise, when high accuracy is a strict requirement, torque measurements are needed. Alternatively, tire-less longitudinal forces represent a robust solution, less accurate but with contained outliers and error deviations. An intermediate solution is represented by formulations that exploit known torque repartition to compute longitudinal forces. This improves accuracy without requiring a more complex sensor setup, but under critical operating conditions—when torque distribution is not predictable—the performance of the estimator decreases.

- Wheel steering angle estimation: as accuracy increases, a more detailed knowledge of the steering–suspension system is required. Simplified formulations with constant steering ratios reduce parametrization needs but at the cost of precision, while including steering–suspension kinematic maps and compliance significantly improves accuracy at the expense of higher computational cost and parameter identification complexity.

Table 4 (a): Summary of Lateral/Combined Dynamic-Based Vehicle State Estimators – Performance Evaluation

Observer	Model Acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.	Operating Domain
Single-Track with Linear Pure Lateral Tire Model	3	Sensitive	Intermediate to IMU and Wheel Speed	Low	1–2	3	Unstable at Low Velocity; Not Reliable in Transient Dynamics
Single-Track with Nonlinear Pure Lateral Tire Model	4	Sensitive	Intermediate to IMU and Wheel Speed	Medium	1–2	3	Unstable at Low Velocity; Not Reliable in Transient Dynamics
Single-Track with Combined Tire Model	4.1	Sensitive	Intermediate to IMU and High to Wheel	Medium	1–2	3	Unstable at Low Velocity; Not Reliable in Transient Dynamics
Double-Track with Pure Lateral Tire Model and Tire-Less Estimation of Longitudinal Forces (Torque Repartition)	4.2	Sensitive	Intermediate to IMU and Wheel Speed	Medium	1–2	3	Unstable at Low Velocity; Less Suitable with Adaptive Logic; Not Reliable in Transient Dynamics
Double-Track with Tire-Model-Based Longitudinal Forces (Wheel Speed)	4.3	Sensitive	Intermediate to IMU and High to Wheel	Medium	1–2	3	Unstable at Low Velocity; Less Suitable with Adaptive Logic; Not Reliable in Transient Dynamics
Double-Track with Tire-Model-Based Longitudinal Forces (Torque Repartition)	4.4	Sensitive	Intermediate to IMU and Wheel Speed	Medium	2	3	Unstable at Low Velocity; Less Suitable with Adaptive Logic; Not Reliable in Transient Dynamics
Double-Track with Tire-Model-Based Longitudinal Forces (Torque Measurements)	4.5	Sensitive	Intermediate to IMU and Wheel Speed	Medium	3	3	Unstable at Low Velocity; Less Suitable with Adaptive Logic; Not Reliable in Transient Dynamics
Double-Track with Steering-Suspension Kinematic Maps	5	Sensitive	Intermediate to IMU and Wheel Speed	High	2–4	4	Unstable at Low Velocity; Less Suitable with Adaptive Logic; Not Reliable in Transient Dynamics
Tire-Less Single-Track Axle Forces W/O Sideslip Angle Estimation	2	Insensitive	High to IMU	Low	1–2	2	Not Reliable in Transient Dynamics and High Load Transfer

Table 4 (b): Summary of Lateral/Combined Dynamic-Based Vehicle State Estimators – Performance Evaluation

Observer	Model Acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.	Operating Domain
Tire-Less Single-Track Axle Forces with Sideslip Angle Estimation	2	Sensitive	High to IMU and Intermediate to Wheel Speed	Low	1–2	3	Reliable Only in Linear Range
Cascade Estimation with Linear Cornering Stiffness Adaptation	3.1	Tunable	High to IMU And Intermediate to Wheel Speed	Low	1–2	3	Reliable Only in Linear Range; Unobservable in Straight-Line Maneuvers
Grip Estimation in Nonlinear Range	4.1	Tunable	High to IMU and Intermediate to Wheel Speed	Medium	1–2	3	Unobservable In Linear Range
Multiple-Model for Road Estimation	4–4.9	Sensitive	Intermediate	Very High	1–4	3	Not Able to Capture Intermediate Road Conditions
Dual Filter	4–4.9	Tunable	Intermediate	High	1–4	3	Unsuitable for Fast Parameter Variation
Joint Observer	4–4.9	Tunable	Intermediate	High	1–4	3	Observability Issues in Low-Excitation Conditions

For simultaneous estimation of multiple parameters and state variables, both accuracy and the increment of computational cost depend on the number of parameters and on the chosen model, but the selection of the observer type mainly depends on the specific identification task. Multiple-model approaches are best suited for discrete road classification tasks, as they guarantee stability but at the cost of high computational demand and limited flexibility. Dual-filter strategies are typically applied for accurate auto-calibration of constant parameters, prioritizing stability over convergence speed. Joint observers are instead employed for onboard detection of fast-varying parameters, though they require simplified models to preserve observability.

6.3 Ride Applications: Estimators of Vertical Dynamics

Road profile estimation plays a key role in modern vehicle control systems, which are increasingly oriented toward preserving passenger safety and comfort, as well as ensuring road holding over surface irregularities.

To substitute the expensive estimation procedures employing profilometers or lidar-based image-processing techniques, various methodologies have been proposed in the literature based on suspension dynamic response, often relying on simplified modeling assumptions to facilitate integration within control architectures and to reduce the number of required measurements. In this regard, key states involved in ride and comfort estimation—such as vehicle body orientation, center of gravity height, or wheel center position—are difficult to measure directly; others, such as suspension deflection, are not always available; others, like the road elevation itself, are generally inaccessible without high-end sensors. This highlights the need, in observer design, to balance the number of state variables and estimation accuracy with the availability of onboard measurements required to achieve full (or local) observability of the system.

The simplest approaches rely on model-based transfer functions, aimed at processing acceleration signals in the Fourier or Laplace domain. In (Liu et al., 2020) the road profile is estimated from the vertical acceleration of the sprung mass using a quarter-car model, where the dynamic relationship between road input and chassis acceleration is described by a fourth-order transfer function. A PSD-based analysis is then applied to classify road roughness levels. The method requires only a single accelerometer mounted on the vehicle body and demonstrates high robustness with respect to variations in sprung mass and damping. In (González et al., 2008), a half-car model with four degrees of freedom is used to estimate the vertical dynamics of the vehicle body and axles, including pitch motion. Road roughness is estimated through calibrated transfer functions linking road elevation to vertical accelerations measured at the vehicle body, enabling classification in accordance with ISO standards. The sensor setup includes accelerometers placed on both the front and rear of the chassis.

These approaches are advantageous due to their minimal sensor requirements and low computational cost. However, they are highly sensitive to sensor noise and require pre-processing of acceleration signals before being used as inputs to the transfer function. Furthermore, they do not provide a direct time-domain reconstruction of the road elevation, but only a statistical characterization over a time window. As such, while they are well-suited for road classification tasks, they are not accurate in capturing localized or instantaneous variations in road elevation, such as those caused by potholes or local road discontinuities (Nguyen et al., 2019).

To combine dynamic models with real-time estimation of road elevation, many approaches rely on state observers,

whose degree of nonlinearity depends on the chosen vehicle-suspension model. In these formulations, road disturbances are typically treated as unknown inputs or as augmented state variables to be estimated online.

In Kalman filter-based architectures, the road height or its time derivative is often modeled as an unknown Gaussian state variable, with its uncertainty defined through the associated process noise. In (Doumiati et al., 2011), the road height time derivative is included in the state vector as a random-walk variable, allowing its estimation simultaneously with the vehicle-suspension states. A linear quarter-car model is adopted, enabling the use of a standard linear Kalman Filter. The measurements include vertical body acceleration and suspension deflection. However, to ensure full system observability, the vertical position of the vehicle's center of gravity was also required, and was obtained via double numerical integration of filtered acceleration signals. This additional step introduces sensitivity to sensor noise and drift, potentially affecting estimation accuracy. To reduce the sensitivity of quarter-car models to local load variations resulting from vehicle load transfer, an additional block is integrated into the estimator architecture in (Yu et al., 2013), where a recursive least squares (RLS) algorithm is employed to adapt the sprung mass parameter of the linear quarter-car model. The approach demonstrates rapid convergence within a short time window.

In (Qin et al., 2017), an adaptive estimation scheme combining a Kalman filter with a super-twisting observer (AKF-ASTO) is proposed for road profile reconstruction using a linear quarter-car model. Road disturbances are modeled as Gaussian unknown inputs. The Kalman filter covariance matrices are adaptively tuned based on road roughness classification, obtained through wavelet analysis. The estimated vehicle response is then processed by the super-twisting observer to reconstruct the road profile in the time domain. Although the approach is more complex, it demonstrates improved robustness and accuracy over fixed parameter KF approaches across varying road conditions.

Alternative procedures have been implemented to address the KF sensitivity to uncertainty on the noise parameters and to further reduce computational complexity.

In (Nodeh et al., 2021) a reduced-order observer is proposed to avoid modeling the road input explicitly as a stochastic variable. Instead, the approach employs an unknown input decoupling framework, combined with a novel Degree of Observability (DO) metric to select the most informative output set. In (Tudon-Martinez et al., 2014) an H_∞ observer is designed based on a linear quarter-car model, where the road disturbance is modeled as an unknown input. The observer estimates suspension states from sprung mass acceleration and suspension deflection measurements, and reconstructs the road elevation by supposing static equilibrium conditions on the unsprung mass. The method proves robustness against varying road conditions. However, the reconstruction of the road profile relies on the assumption of static equilibrium, potentially limiting its accuracy during transient or highly dynamic maneuvers. Moreover, tuning the H_∞ weighting functions to balance robustness and responsiveness makes the estimator accuracy dependent on the implementation choices, as well as in KF-based logics.

Recent works have extended road profile estimation beyond linear quarter-car assumptions by employing half-car models and suspension system nonlinearity. These adjustments increase model reliability while requiring more complete sensors' equipment (in case of half-car modeling), including pitch rate measurements or multiple accelerometers, and have to be implemented in more computationally demanding and nonlinear filters.

In (Zhu et al., 2022), a Kalman filter-based estimator is developed using a detailed half-car model, where the road disturbance is estimated directly through a joint input-state framework. It is important to specify that the accuracy of road profile estimation based on half-car models is influenced by vehicle speed. To mitigate this effect, the method adaptively adjusts the sampling rate according to vehicle velocity to maintain constant spatial resolution.

More recent works include in the modeling formulation, variable suspension parameters arising from control strategies, nonlinear suspension constraints, and bushing compliances. These additional features require time-varying parameter state observers, as in (Pham et al., 2021), where a nonlinear parameter-varying rheological suspension is modeled, and in (Zhao et al., 2023), where a reduced-order bushing modeling and control arm constraints are incorporated in the quarter car formulation.

In (Du et al., 2020), a nonlinear extended state observer (NLESO) is designed for a half-car with an active suspension system, explicitly modeling suspension nonlinearities and treating disturbances as extended states.

6.3.1 Ride Applications: Comparative Analysis

In Table 5, the described methodologies are summarized, without including the operating domain, which is equivalent for all the algorithms. The current state of the art in ride-oriented estimation remains largely limited to simplified formulations, mostly relying on quarter- or half-car models with approximate representations of suspension dynamics. Implementation of more reliable modeling approaches, while maintaining real-time capability, remains a current open research field.

Transfer-function-based methods, with minimal sensor requirements, are particularly suitable for road roughness classification, where statistical characterization is sufficient and computational simplicity is prioritized. Conversely, Kalman Filter-based and unknown-input observers, employing a linear quarter car model, aim at the online

recognition of single asperities modeled as stochastic variables; however, their broader applicability is constrained by the need for suspension deflection measurements. Furthermore, their robustness depends on the uncertainty associated to the road height variation, which is defined by the filter parameters, and employing adaptive filter strategies increases computational cost.

Although these simplified formulations are easily integrated into control systems, they neglect nonlinear effects such as corner load transfer, suspension parameter variability, and elasto-kinematic constraints. Accounting for these phenomena significantly improves estimation accuracy but comes at the cost of increased computational burden and filter nonlinearity, and requires more complete parametrization of the system.

7. Hybrid State Estimators: Kinematic-Dynamic Fusion Logic

As outlined in the previous sections, tire-model-based estimators for handling applications decrease in their reliability during transient maneuvers or under highly nonlinear conditions, since the implemented tire models are generally valid in steady-state conditions. The use of more complex tire models can extend their validity range, but at the cost of requiring highly nonlinear filtering schemes, which compromise computational efficiency.

Table 5: Summary of Vertical/Ride Dynamics Estimators – Performance Evaluation

Observer	Model Acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.
Quarter-Car TF	2.0	Insensitive	High to Accelerometer	Low	2	2
Half-Car TF	2.1	Insensitive	High to Accelerometer and Rotational Speed	Low	2	2
Linear Quarter-Car KF	2.2	Sensitive	Intermediate to Accelerometer and Spring Deflection	Low	3	2
Linear Quarter-Car KF (Sprung Mass Adaptation)	2.3	Sensitive	Intermediate to Accelerometer and Spring Deflection	Medium	3	2
AKF + ASTO (Quarter-Car)	2.4	Tunable	Intermediate to Accelerometer and Spring Deflection	Medium	3	2
Unknown-input Decoupling / H_∞ (Quarter-Car)	2.2	Sensitive	Intermediate to Accelerometer and Spring Deflection	Low	3	2
Half-Car KF	2.3	Sensitive	Intermediate to Accelerometer, Rotational Speed and Spring Deflection	Low	3	2
Quarter-Car with Nonlinear Suspensions	2.5	Sensitive	Intermediate to Accelerometer and Spring Deflection	Medium	3	2
Half-Car with Nonlinear Suspensions	2.6	Sensitive	Intermediate to Accelerometer, Rotational Speed and Spring Deflection	Medium	3	5
Quarter-Car with Nonlinear Suspensions Including Bushings	2.7	Sensitive	Intermediate to Accelerometer and Spring Deflection	Medium	3	5

On the other hand, kinematic estimators suffer from loss of observability in straight-line driving and are more sensitive to accelerometer errors. To address these limitations, hybrid approaches are proposed in literature to combine kinematic and tire-model-based logics, leveraging the strengths of both while mitigating their respective weaknesses.

To simply address the observability problem during straight-line driving, in (Galluppi et al., 2018), the kinematic approach is modified by introducing an additional measurement equation. The lateral velocity estimated by the kinematic model in the time-update equations is used to compute the lateral acceleration through a single-track vehicle model; this acceleration output is then compared with the measured value to compute innovation within a Kalman filter framework and correct the estimation. To reduce sensitivity to parameter uncertainty, the approach was further extended by incorporating real-time friction coefficient estimation (Carnier et al., 2023). However, in this approach, the weighting between the kinematic and dynamic contributions is entirely managed through the covariance matrices, without explicitly accounting for the type of maneuver and the consequent higher reliability of one formulation over the other.

To improve accuracy during transient maneuvers, some hybrid estimators combine the estimators' results using a weighting function designed as a complementary filter, where the kinematic formulation dominates at higher frequencies and the dynamic formulation prevails at lower ones. In (Piyabongkarn et al., 2008), this weighting scheme relies on fixed parameters tuned offline.

Alternatively, the fusion can be performed through time-varying weighting strategies, where the contribution of each estimator is continuously adapted according to the current driving scenario.

In (Oh & Choi, 2011), a transient factor is used to combine the lateral velocity estimations from a single-track-based Luenberger observer and from a kinematic-based numerical integration. The factor is computed from normalized indicators of lateral acceleration, yielding a scalar value that increases with maneuver aggressiveness.

An alternative methodology was proposed in (Rezaeian et al., 2016), where the weighting function is based directly on yaw-rate-related observability to manage the estimation between a kinematic estimator and an inverse-model-based observer. The latter consists of a tire-less observer to predict lateral forces and an inverse tire model to estimate the tire sideslip angle, which is then transferred to the vehicle's center of gravity. This dynamic approach is not reliable under nonlinear conditions, which, however, are characterized by high variation of yaw rate, allowing the kinematic estimator to have a dominant estimation contribution in such cases.

To optimize performance, by simultaneously taking into account all the aspects related to accuracy and observability domains, more detailed fuzzy logic approaches rely on leveraging the estimation results between the different models based on the combination of multiple measured or observed variables.

In (Xia et al., 2022), the output of a single-track-model-based consensus Kalman filter is combined with that of a kinematic observer for vehicle sideslip angle estimation. The fusion is performed through weighting functions that depend on a combination of yaw rate, to account for the observability of the kinematic observer, and steering angle, together with lateral acceleration, which indicate the instantaneous level of maneuver nonlinearity. In (Xia et al., 2021), a vehicle sideslip angle estimation method is proposed by fusing a kinematics estimator using single-antenna GNSS measurements, with a dynamics-based augmented Kalman filter. Observability under low-excitation conditions and accuracy are improved through a fuzzy logic module that uses as inputs the yaw rate, vehicle lateral acceleration and the estimated errors from both filters.

To maximize the accuracy of the fusion logic with respect to a fully known vehicle, some architectures employ artificial intelligence to perform a kinematic dynamic fusion strategy, such as in (Lee et al., 2024), where the outputs of a kinematic and a dynamic vehicle model are fused for sideslip angle estimation through a neural network-based weighting strategy. The network, trained offline on experimental data, takes as inputs yaw rate, lateral and longitudinal acceleration, steering angle, and the difference between the two model estimates, and outputs the optimal fusion coefficient. This data-driven approach allows the weighting to adapt in real time to varying driving conditions, but it requires the availability of data representative of the entire domain in which the vehicle operates.

Another strategy, proposed in (Chen et al., 2023) introduces three parallel observers: a kinematic one, primarily operating during transient maneuvers; a linear tire-model-based observer, optimized for efficiency and robustness under steady-state, linear conditions; and a nonlinear tire-model-based observer, which, while still unsuitable for transient dynamics, remains robust under nonlinear operating conditions and achieves higher accuracy than the kinematic observer in such cases. Their outputs are combined through a fusion logic that adaptively weights each contribution according to the current side slip angle range, taking into account the higher accuracy of the nonlinear tire-model-based approach compared to the kinematic one. This architecture improves estimation reliability across a broader range of maneuvers, at the cost of increased computational load due to the simultaneous execution of three observers.

Literature works implement fuzzy logic in adaptive estimation strategy, such as in (Li et al., 2014), where an extended Kalman filter-based kinematic estimator incorporates, as an additional measurement, the lateral velocity estimated by a dual extended Kalman filter implementing a single-track vehicle model and performing real-time estimation of tire model parameters. A fuzzy decision module employs the instantaneous longitudinal velocity, to account for the reduced stability of the dynamic model at low speeds, and the yaw rate, to address the unobservability of the kinematic estimator.

In (Xiong et al., 2019) the fusion between a kinematic estimator and a single-track-model-based dynamic observer also includes the estimation of sensor biases and roll/pitch angles, based on integration of pitch and roll rate signals and on the discrepancies between kinematic and dynamic estimations. In (Liao & Borrelli, 2019), an alternative adaptive procedure is proposed, in which the kinematic and dynamic strategies operate independently. The kinematic observer, incorporating online compensation for road gradient effects, provides a preliminary sideslip angle estimate. This estimate is used in a recursive least squares-based cornering stiffness estimator, whose output is then provided to a dynamic observer. Although the approach relies on a less accurate estimation, it improves robustness by

decoupling the two observers, thereby enhancing stability since the error of one does not propagate to the performance of the other.

7.1 Hybrid Estimators: Comparative Analysis

Table 6 summarizes the pros and cons of the described hybrid virtual sensing methodologies. The operating domain is not included because these techniques are designed to cover all the vehicle operating conditions. Compared to purely dynamic observers, which remain inherently sensitive to parameter variations unless adaptive logics are integrated, hybrid approaches mitigate this issue by combining a sensitive formulation with an insensitive one, thus allowing robustness to be tuned through the weighting logic. Architectures where the fusion weights are optimized via AI-based strategies may be more vulnerable to parameter variations, since their reliability strongly depends on the dataset used for training (Lee et al., 2024).

The hybrid estimator accuracy is strongly linked to the underlying dynamic formulation—particularly in steady-state conditions—but also to the complexity and detail of the fusion logic: the more effectively the logic captures the critical conditions in which dynamic models lose accuracy, the more reliable the estimator becomes.

Table 6: Summary of Hybrid Kinematic–Dynamic Estimators – Performance Evaluation

Observer	Model Acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.
Kinematic + Single-Track KF	4.0	Tunable	Intermediate to IMU and Wheel Speed	Low	1	3
Fusion Based on Weighting	4.3	Tunable	Intermediate to IMU and Wheel Speed	Medium	1	3
NN-Based Fusion	4.5	Sensitive	Low	High	1	3
Triple-Observer Fusion	4.5	Tunable	Low	High	1	3
Dual EKF + Fuzzy	4.5	Tunable	Intermediate to IMU and Wheel Speed	High	1	3
Kinematic + Single-Track + Bias/Orientation	3.5	Tunable	Intermediate to IMU and Wheel Speed	Medium	2	3
Kinematic + Single-Track + Bias/Road Gradient and Stiffness	3.5	Tunable	Intermediate to IMU and Wheel Speed	Medium	2	3

However, as the fusion logic or models grow in complexity, the computational burden increases significantly, especially when multiple dynamic observers are executed in parallel (Chen et al., 2023) or when parameter adaptation of nonlinear models is included in the estimation.

An intermediate trade-off between accuracy, robustness, and efficiency is achieved by employing simplified dynamic models—typically single-track linear formulations—augmented with parameter adaptation, such as cornering stiffness or road gradient estimation for signal compensation (Liao & Borrelli, 2019; Xiong et al., 2019). These maintain a more linear and robust structure while still accounting for additional physical effects, thus balancing computational cost with reliability across a wide range of operating conditions.

8. State Observers Employing Multibody Models

Multibody-based state estimators are an increasingly relevant research topic, with ongoing efforts focused on enabling their real-time implementation and incorporating the inherent nonlinearities of such models into state observers (Sanjurjo et al., 2017). Unlike the simplified approaches discussed in previous sections—which represent the vehicle either as rigidly connected to the wheels or with idealized suspension models based on massless linear or nonlinear springs—multibody-based estimators provide a more reliable physical representation and can estimate a broader set of internal state variables. This includes, in particular, suspension kinematics and dynamics, as the relevant constraints are explicitly embedded in the model.

Due to their computational complexity, arising from the large number of state variables, dedicated reduction strategies are required to meet real-time constraints. Some works in the literature have explored data-driven approaches, in which detailed offline multibody simulations are used to generate training datasets for tuning surrogate

and non-explicit models. In (Cuesta et al., 2016), a non-explicit multibody formulation was applied to a single suspension system, defining a data-driven nonlinear mapping between accelerations and the instantaneous position, velocity, and external force level states. In (Pan et al., 2021; Ye et al., 2021) deep learning architectures were trained on multibody simulation data to learn state-evolution functions. However, these approaches rely on a sufficiently rich and representative training dataset to prevent overfitting to a restricted domain of operating conditions.

As an initial case study incorporating an explicit multibody formulation of a full vehicle, (Cuadrado et al., 2012) implements a high-fidelity multibody vehicle model with rigid suspension constraints in an Extended Kalman Filter, adopting a Lagrangian formulation reduced via the matrix-R method. The state vector comprises chassis motions, suspension deflections, and wheel rotations, while the measurement set includes chassis translations and rotations together with suspension vertical displacements. Experimental validation under dynamic maneuvers confirms the accuracy of the approach, but with a computational burden exceeding real-time feasibility.

To address this critical issue, (Cuadrado et al., 2013) proposed a more efficient implementation by modeling each suspension as a macro-joint within a multibody formulation in relative coordinates. The kinematics of these macro-joints are precomputed and stored in lookup tables, which map suspension travel (and steering angle for the front) to the other constrained variables and their partial derivatives. The method significantly reduces computational cost while retaining high-fidelity suspension kinematics, and supports real-time execution, even in hardware-in the-loop simulations, as demonstrated in (Parra et al., 2020).

In (Sanjurjo, 2016), the same efficient macro-joint multibody formulation was applied and validated on simulation data for a complete vehicle equipped with front and rear double-wishbone suspension systems, employing a nonlinear tire model to estimate ground forces. The estimation architecture relied on an error-state EKF and incorporated measurements from IMU-recorded accelerations, GPS position and velocity, steering angle, suspension deflections, and chassis angular velocities measured by gyroscopes. Sensor signals were generated from simulation channels with added white noise to emulate realistic conditions. This framework was later extended in (Rodríguez et al., 2021), into a Dual Kalman Filter configuration capable of estimating, in real time, both the chassis mass and the road friction coefficient.

8.1 State Observers Employing Multibody Models: Comparative Analysis

In Table 7, the progressive improvements achieved for this estimation category are reported, underlying the improvement in accuracy and efficiency. However, various limitations still characterize the current state of the art.

First of all, the reported logics have been designed under the assumption of a flat road profile, without explicitly accounting for road-induced excitations. Consequently, their application to ride comfort-oriented estimators remains an open research field.

Table 7: Summary of Multibody-Based Vehicle State Estimators – Performance Evaluation

Observer	Model Acc.	Robustness	Sensors' Influence	Comp. Cost	Sens. Req.	Param. Req.
Data-Driven Surrogate Multibody Models	6.0	Sensitive	Low	Very High	5	5
Full Explicit Multibody EKF (Matrix-R Reduction)	6.1	Sensitive	Low	Very High	5	5
Macro-Joint Reduced Multibody-Based Estimator	6.1	Sensitive	Low	High	5	5
Dual Kalman Multibody with Mass and Friction Adaptation	6.2	Tunable	Low	High	5	5

Furthermore, their main limitation is the reliance on detailed knowledge of system parameters—masses and inertias of single rigid elements, and geometric properties—whose identification is complex. Therefore, a key open challenge for future developments is the design of reduced-order models that simplify parameterization, both for offline identification and for use in self-tuning algorithms.

9. Conclusions and Further Perspectives

This paper has presented a comprehensive review of model-based vehicle state estimation techniques, analyzing 200 articles classified into four main categories: kinematic, vehicle- and tire-model-based, hybrid, and multibody

approaches. Methodologies covering multiple applications have been discussed and compared, including handling under pure longitudinal, lateral, and combined dynamics, and ride-comfort applications. Furthermore, the integration of adaptive strategies has been examined. The reviewed works were selected among peer-reviewed conference and journal papers, including contributions with both general and detailed key findings, aimed at providing innovation with respect to the state of the art at the time of publication and supported by either experimental validation or high-fidelity simulation data.

For each category, modeling assumptions, observer architectures, and sensor requirements have been analyzed, highlighting their respective strengths, limitations, and domains of applicability. The comparative analysis confirms that no single approach is universally optimal: the most suitable choice depends on sensor availability, desired accuracy, level of parametrization, and computational constraints. Accordingly, practical guidelines have been defined to support the selection of virtual sensing strategies tailored to specific application requirements, based on objective performance metrics.

Based on these indicators, kinematic observers represent the optimal choice when minimizing parametrization requirements and when sensitivity to environmental conditions is a priority. Vehicle- and tire-model-based observers are suitable for enhancing accuracy and enabling estimation of tire-related variables with intermediate sensor setups and incomplete vehicle–tire parameterization. In this regard, adaptive strategies and hybrid formulations are necessary to improve robustness and stability in transient dynamics. Multibody approaches provide the highest accuracy but can only be adopted when vehicle parametrization also includes detailed suspension–steering system knowledge and dedicated sensor and hardware equipment. Finally, low-level implementation guidelines have been provided for the variable applications.

Further research is focused on addressing current limitations and extending the applicability of state observers to a wider operating domain. Promising directions include the integration of additional physical effects often neglected in simplified models, the exploitation of emerging sensor technologies to improve observability of unknown variables, the development of reduced-order formulations enabling real-time implementation and parametrization of high-fidelity models, and the design of hybrid schemes that merge model-based observers with AI-driven strategies. These advancements are expected to enhance both the robustness and reliability of vehicle state estimation and their large-scale applicability.

A key outcome of this review is that tire-model-based estimators, while essential to compensate for the intrinsic limitations of kinematic approaches, exhibit performance strongly dependent on parameter variability due to tire aging, thermal effects, and wear. Although adaptive estimation strategies can partially mitigate these effects, they still require convergence time and careful observability analysis. A promising research direction lies in integrating multiphysical influences on tire mechanics—such as thermal and wear effects—into estimation frameworks. In simulation scenarios, the benefits of co-simulating thermal and wear models with dynamic tire formulations have already been demonstrated. The extension of this methodology to real-time observers is a promising research open field, which would enable global estimators capable of simultaneously performing online adaptation of tire parameters, while estimating their thermal state and wear level, improving observability of tire mechanical property changes. In this regard, (Longobardi et al., 2025) implements a real-time state observer including a tire thermal model. For wear estimation, current solutions mainly rely on machine learning algorithms or intelligent tire systems (Kim et al., 2023; Li et al., 2021).

In this regard, future advances in virtual sensing are expected to be strongly influenced by the development of emerging technologies, and in particular of the intelligent tires (Yang et al., 2022). These systems integrate sensors directly within the tire structure, such as accelerometers or strain gauges, enabling the direct measurement of quantities like deformation (Mendoza-Petit et al., 2019), contact forces (Xu et al., 2020), or slip that are generally estimated from encoders or chassis-mounted sensors through model-based observers, as discussed in this article. By providing such direct measurements, intelligent tires can reduce the number of unknown variables in the estimation process involving model-based logic. This not only improves the precision of the measured quantities themselves but also enhances overall system observability through the additional measurements provided, which is particularly beneficial in adaptive estimation schemes aimed at identifying parameters that vary over time, such as tire wear and road surface conditions (Pomoni, 2022). Although currently limited to advanced research applications in autonomous driving—where their effectiveness has already been demonstrated—ongoing research and development from tire and bearing manufacturers aim to extend the applicability of intelligent tires to mass production contexts (Singh et al., 2019).

As already discussed in multibody-model-based approaches, a promising research direction concerns the integration of steering and suspension dynamics within vehicle state estimators, in which, as previously described, dedicated reduction techniques have been developed to make the models suitable for embedded implementation. However, their practical adoption is still hindered by the need for a highly detailed parameterization of the suspension–steering system, a level of detail that is generally not obtainable solely from experimental data. A current research

trend, already explored in simulation contexts, is the design of reduced-order models that preserve the accuracy of full multibody formulations while requiring a significantly smaller set of parameters (Cuadrado et al., 1997; Wu & Tiso, 2016). This simplification facilitates parameter identification both offline—using indoor measurement campaigns—and online, through self-calibration procedures.

Another relevant future development concerns the application of artificial intelligence (AI) techniques to virtual sensing, which, as previously specified, is a task of growing interest. In this context, data-driven approaches have demonstrated a high level of accuracy in several virtual sensing applications, including wheel dynamics estimation, longitudinal vehicle speed estimation, and sideslip angle estimation (Marotta et al., 2024; Kuutti et al., 2020). In each case, detailed descriptions of data collection and preparation are typically provided to ensure the robustness of the algorithm. However, as a consequence, their performance is strongly dependent on the quality and representativeness of the training dataset, which limits robustness when operating conditions or system parameters vary beyond the training domain. Current research, therefore, focuses on combining AI-based estimators with physics-based models, leveraging the adaptability of data-driven approaches while retaining the generalization and interpretability offered by model-based logic. In particular, hybrid approaches combining physical-based state observers and neural networks—either by using neural networks to identify or adapt state–space model parameters for the state observer, or by updating neural network parameters via Kalman filtering—have shown superior accuracy and generalization compared to either method alone, especially in nonlinear systems (Feng et al., 2023). Furthermore, Physics-Informed Neural Networks (PINNs) represent an emerging solution that integrates physical laws directly into the learning process (Tan et al., 2023) reducing the need for large datasets and improving robustness when extrapolating beyond the training domain.

Future developments of this review will include extending the general assessment of estimation strategies toward a more structured performance benchmarking, grouping methods by application type and testing them under comparable conditions using synthetic data. While validation based on experimental measurements is generally more reliable and representative of real operating conditions, the use of synthetic data—generated within fully editable simulation frameworks—allows for an objective evaluation of estimator sensitivity to parameter variations, model simplifications, different noise levels, and diverse sensor setups. Moreover, this approach would enable the objective investigation of how different operating scenarios influence the reliability and robustness of each estimation approach.

10. Fundings

This work was supported by the project “Homo-AD: Vehicle and Passenger oriented holistic motion planning for Autonomous Driving” funded by the Italian MIUR “Progetti di Ricerca di Rilevante Interesse Nazionale (PRIN)”–CUP E53D23017110001.

11. Acknowledgments

The authors gratefully acknowledge the editorial office of Reports in Mechanical Engineering for granting a full waiver of the Article Processing Charge (APC).

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