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Case Study on Mechanical and Operational Behavior in Steel Production: Performance and Process Behavior in Steel Manufacturing Plant

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ABSTRACT

The steel production industry plays a vital role in global industrialization, driving infrastructure development and economic growth. Despite its importance, the sector faces persistent challenges such as operational inefficiencies, fluctuating demand, environmental constraints, and reliability issues within mechanical and production systems. This study examines steel manufacturing processes from a mechanical engineering perspective, analyzing system dynamics, equipment performance, and process reliability through both quantitative and qualitative approaches. Key operational patterns, failure points, and opportunities for process optimization are identified. The results reveal that integrating advanced mechanical systems, predictive maintenance strategies, and optimized workflow designs can significantly enhance reliability, reduce downtime, and improve overall process efficiency. These insights offer practical guidance for engineers and plant managers aiming to strengthen the mechanical resilience and sustainability of steel production operations.

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1. Introduction

The steel industry is essential to contemporary industrialization, providing crucial material for the building, automotive, and equipment sectors. Notwithstanding its significance, the sector has intricated behavioral patterns shaped by economic cycles, regulatory demands, and technology improvements. Comprehending these behaviors is essential for stakeholders to make educated decisions about production, investment, and sustainability.

The steel industry involves the processing of iron ore into steel and the subsequent transformation of that metal into finished products. The entire process encompasses several entities, including manufacturers, suppliers, transporters, warehouses, retailers, and customers, where operations research methods are employed to achieve an efficient and successful industry, minimizing costs and maximizing revenues. The issues confronting the steel industry in the era of globalization are intricate; the key to a lasting turnaround resides in how the sector addresses these challenges and cultivates competitive and anticipatory processes. Most of the issues may be addressed by adopting and adjusting their operational management approach. Over 50% of positions within the industry pertain to customer service, quality assurance, production planning, scheduling, inventory management, and logistics. Furthermore, all other functional areas are interconnected with these roles, where operations research and their diverse tools have demonstrated efficacy.

Structural reliability analysis examines the safety of a structure in relation to numerous influencing variables. It is, in fact, the likelihood of the structural system fulfilling the designated purpose. Numerous methods exist for calculating reliability (Diamantidis et al., 2025; Ellingwood et al., 2025; Mai et al., 2018); XIE and ZHOU (Xie et al., 2011; Zhou, 2010; Zhou & Xie, 2009) examined the generating function approach in structural reliability analysis. Subsequently, CHANG employed the generating function to evaluate the precision of some theoretical limits inside the relaxed linear programming boundaries approach (RLP) (Chang & Mori, 2013; Heidweiller & Vrouwenvelder, 1988; Song & Der Kiureghian, 2003; Yu et al., 2020). This study examines the dependability analysis of steel structures. Brittle fractures of steel buildings were observed during the Kobe earthquake in 1995 and the Northridge earthquake in 1994. The oversight of brittle failure in steel has been a significant shortcoming in previous studies. This research proposes a novel way for calculating the dependability of structural systems. The suggested approach can estimate a steel structure with many elements. During the construction of steel structures, researchers frequently focus solely on traditional plastic failure (Ferreira Filho et al., 2024; Shefeng et al., 2011; Yu et al., 2020). Brittle failures were seen at the steel beam-column junctions during the Northridge earthquake in 1994 and the Kobe earthquake in 1995. Consequently, both plastic failure and brittle failure must be considered in the safety analysis of steel buildings.

With a projected worldwide turnover of 900 billion USD (World, 2015), the steel industry is the world's second-largest behind the oil and gas sector. Building and construction, packaging, transportation, and renewable energy are just a few of the numerous industries that rely on steel. The output of crude steel more than quadrupled in the previous 30 years, reaching 1,665 million metric tons in 2014. The steel production industry is bound to be held responsible for environmental responsibilities due to its productivity. This sector accounts, for instance, for 12% of the CO_2 emissions in nations like China (Li et al., 2016). Therefore, it is critical to examine the steelmaking processes to illustrate the primary environmental effects and potential remedies through the adoption of a circular economy model.

The first stage in the process of attempting to "close the loop" of product life cycles (including that of steel) through increased recycling and re-use (Notarnicola et al., 2016) is to conduct an efficient and methodical analysis of such product systems in terms of their impact on the environment using techniques such as Life Cycle Assessment (LCA) (Notarnicola et al., 2015). A method of this kind has been created, and in certain instances, it has been embraced as a key instrument for particular policies, such examples of which include the Integrated Product Policy of the European Union (Suer et al., 2022). Specifically, life cycle assessment (LCA) makes it possible to evaluate a product system from an environmental perspective by taking into account all phases of the product's life cycle in a comprehensive manner. These stages include everything from the extraction of raw materials to the disposal of the product in its final form. Historically, this kind of instrument has been utilized for the purpose of assessing the environmental performance of steel product systems. Throughout the course of the previous few decades, the World Steel Association (Ebrahimi & Koh, 2021) has commissioned three Life Cycle Inventory (LCI) studies that have focused on the manufacture of 15 different steel products. These items include plates, coils, rods, and pipes. The most current research was conducted in 2010, and the statistics are generally cradle-to-gate, which includes recycling at the conclusion of the product's manufacturing life. The manufacturing procedures that are being evaluated include those of the traditional integrated steel mill as well as the more contemporary ones that make use of direct reduction and electric arc furnaces and are now being utilized in sixteen different steelmaking facilities all over the world. The findings also include some incomplete impact assessment data that was supplied for the purpose of illustration. This data pertains to the lifetime of sections, hot-rolled coil, and hot-dip galvanized steel. The Ilva facility that was briefly mentioned before is not one of the facilities that are taking part in this initiative. An LCI study was conducted by the Eurofer (Hapuwatte et al., 2023) association, which focused on 18 sheet products that were manufactured in seven different European countries (including Italy, but not the Ilva plant, which only produces carbon steel and low alloy non-stainless steel). These sheet products were made with various types of stainless steel and had different surface finishes. Although the research takes into account all stages of the lifespan, from the extraction of raw materials to the gate of the mill, it does not take into account trash that is disposed of in landfills. The analysis is being conducted with the intention of providing stainless steel manufacturers with the opportunity to conduct an environmental benchmarking of their production phases as well as their finished goods. As a method of providing worldwide data for further case studies, the International Stainless Steel Forum (Joachimiak-Lechman, 2024) has also conducted a cradle-to-gate LCI research involving stainless steel goods. This study is a continuation of the European study that was stated before. Long and flat products (austenitic and ferritic grades) that are manufactured from ore and scrap steel are the subject of the data that was obtained from manufacturers in five different countries: Europe, North America, Korea, and Japan. At the beginning of the 1990s, a cradle-to-gate LCI research was conducted on Canadian integrated and scrap-based steel mills (Pinto et al., 2019). This study was conducted prior to the worldwide LCI programs that were discussed before. In the decade that followed, the research was revised and incorporated with information from steel factories in the United States. A simulation of reference plant inventory was carried out by making use of the particular plant data in order to achieve the goal of representing an average condition of the features of electric arc furnaces and integrated mills in the United States of America and Canada. When it was not possible to directly get data from the plants that were being evaluated for this study, statistical data was utilized in order to model the processes and estimate the inventory inputs and outputs. One example of this is the procedure that was used to estimate the emissions to the air (Backman et al., 2019).

The purpose of this research is to develop a mathematical reliability model for use in steel production operations in order to optimize maintenance and forecast system availability. In the past, researchers have investigated a wide range of dependability modelling strategies for industrial systems. These techniques include Markov and semi-Markov methods, hidden-state modelling, and data-driven approaches for predictive maintenance. However, the majority of research has concentrated on component-level models or frameworks that are driven only by data, despite the fact that these methodologies have led to a better understanding of system degradation and failure behavior. The current research endeavors to fill this void by building an integrated mathematical reliability model that establishes a connection between analytical and operational viewpoints. This model is designed to forecast system availability and optimize maintenance methods in steel production settings (Liang et al., 2022; Pan et al., 2024).

The Runge - Kutta technique was selected for resolving the differential equations in our dependability model because of its equilibrium between precision and processing efficiency. In contrast to more rudimentary techniques like Euler's method, the Runge - Kutta method offers superior accuracy while maintaining manageable computing complexity, making it appropriate for systems exhibiting nonlinear or time-varying dynamics. Moreover, it provides enhanced stability and reduced cumulative numerical errors, which are essential for forecasting system availability and optimizing maintenance plans in steel production processes. The manufacturing sector is primarily divided into four components: hot document, cleansing, cutting machines, and sharpening. Individuals are in charge of influencing the system's capacity and accessibility. We now go over the definitions of the different components of symbols, and presumptions needed to create an algebraic structure for plant behavioral assessment.

2. Explanation of the System, Annotations, and Presumptions

i. The grinding process, technology, or subsystems R: -

ii. Cutting is frequently done with sharpening equipment. If this equipment malfunctions, the whole thing may fail severely.

iii. The cleansing of the device, (component S):

iv. For cleaning steel pieces in an ongoing manner beneath exploding discharges at a specific acceleration, cleansing machinery is specifically made. Several band thicknesses can be treated by the aluminum panel descales; the narrow strips range in breadth from 50 to 800 mm, while the massive machinery (S1-S2) operating in tandem range in breadth from 80 to 2100 mm. A single computer can operate that component at a decreased capability.

v. The Hot Steckel Mill, or subsystem's functionality M:

vi. This traditional Steckel steel mill setup comprises of an edger and smoother that work together to roll out chunks of 25 - 45 mm diameter transferable bar. It can operate at less horsepower and is made up of five separate devices interconnected in sequence.

vii. N (the sever Machinery) Sub-system:

The SM-8 and SM-10 are the two primary varieties of cutting machines. Slabs that support anything blossoms, and seats can all be cut in-line with the SM-8 slicing machinery. The exact same cylindrical and perpendicular motors are used in this technology, which is a modification of the SM-10. This is an individual computer with limited functionality. We have additionally utilized additional annotations along with the ones employed for the different sub-systems, namely R, S, k, and N.

i. a: The component is operational and functions minus any problems.
ii. b: preventative care is being performed on the component.
iii. γ: The component in question is being fixed.
iv. : shows how the grinding apparatus is operating in relation to u, whereby u = a, b, along with c.
v. $\mathrm{M}_{c}^{v}{}_{d}^{w}$: shows how the subsystems M_c and M_d are operating in relation to V and W, whereby V and W stand for operational and replacement states, respectively.
vi. Something is capable of being expressed as

$$\mathrm{M}_{c}^{v}{}_{d}^{w} = \begin{cases} \mathrm{M}_{3,4,5,6}^{v}{}_{3}^{w} \\ \mathrm{M}_{3,4,5,6}^{v}{}_{4}^{w} \\ \mathrm{M}_{3,4,5,6}^{v}{}_{5}^{w} \\ \mathrm{M}_{3,4,5,6}^{v}{}_{6}^{w} \\ \mathrm{M}_{3,4,5,6}^{v}{}_{7}^{w} \end{cases}$$

Here $\mathrm{M}_{3,4,5,6}^{v}{}_{3}^{w}$ symbolizes the sub-systems M_4, M_5, M_6, M_7, and the sub-system M_3's restoration status.

Additional icons can be used in a comparable way.

vii. $S^{t\quad y}_{x\quad 3-x}$: shows the sub-systems S_x and S_{3-x} in their corresponding operating states with regard to t and y, wherein t and y are in their repaired state. Something can be expressed as

$$S^{t\quad y}_{x\quad 3-x} = \begin{cases} S^{t\quad y}_{1\quad 2} \\ S^{t\quad y}_{2\quad 1} \end{cases}$$

Where in the world, $S^{t\quad y}_{1\quad 2}$ encompasses both the sub-system's repairing condition and its operational condition S_1, S_2

viii. λ_1: pertains to the subsystems R's steady changeover rate, leading the whole system to enter an altered state.
ix. $X_d(c)$: The components S, M, N, and R's delayed restoration numbers and interrupted times for repairs
x. $da\ \forall da$: 1, 2, 3, ..., 8.
xi. N^u: shows the milling instrument's operating condition in relation to u, whereby u = a and c.
xii. $\phi_d(w)$: The components S, M, N, and R have corresponding rates of failure, wherein d is 1, 2, 3,, 8.
xiii. $\psi_1(v)$: corresponds to the sub-system's rate of routine upkeep (R) lower the elapsed period for repairs (v).
xiv. $\rho_1(t)$: Show that the equipment is operating at maximum efficiency.
xv. $\rho_9(t)$: The likelihood that the equipment has an experienced repair period v and is in condition at moment t.
xvi. $\rho_1(v, w, t)$: Shows the likelihood that the appliance is in condition c at time t, with anticipated period of failure w and prolonged period for repair v = 2,3,......8, 10,11.... 25.

3. Assumptions

For the current examination, the subsequent assumptions were made and put into consideration.

i. There is no connection between disaster and rehabilitation.
ii. The components' breakdown and restoration percentages are regarded as changeable.
iii. For a certain amount of time, a refurbished unit performs as well as original.
iv. Adequate space for repairs is offered.
v. The equipment is capable of functioning undergoing scheduled upkeep.

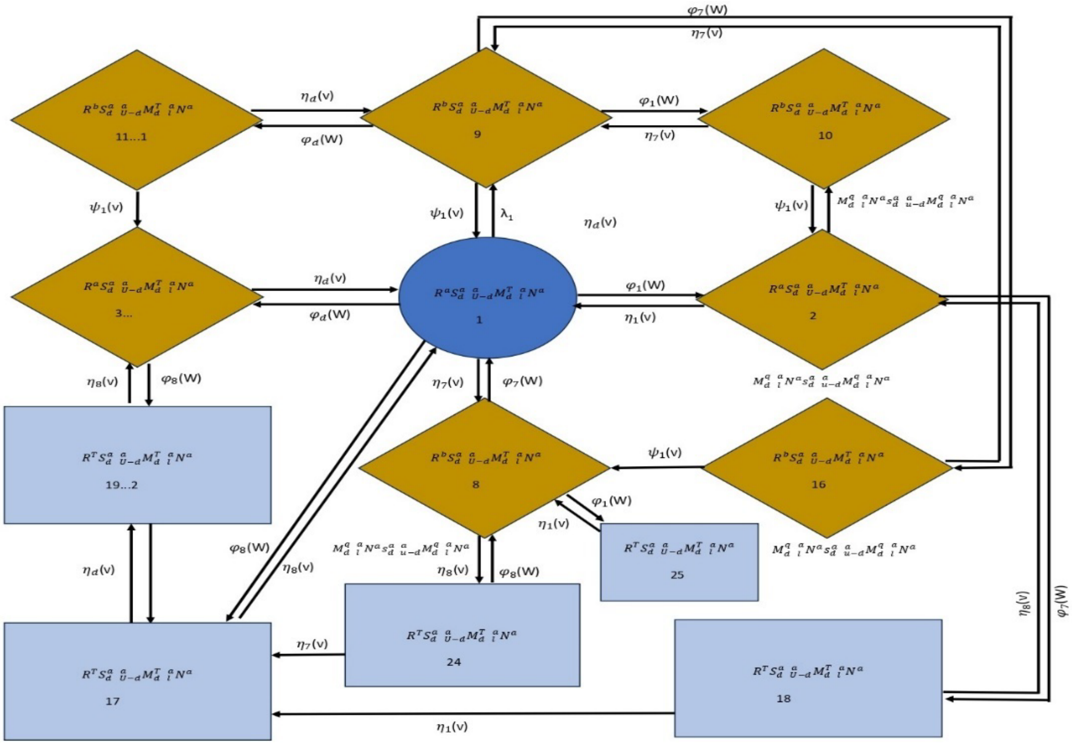

Figure 1: Diagram of the Steel Production Plant's Transition

Unavailability: For repaired infrastructure, accessibility is an operational requirement that takes into consideration a system's maintenance and dependability. The likelihood that the equipment will not be deployed or awaiting maintenance when it is time to use it is known as available. A likelihood ranging from 0 to 1 represents the quantity of availability. The computation of unavailability accounts for both breakdowns and maintenance. For instance, if an illumination has 99.9% access, there will be one out of every thousand occasions where someone has to use it but it has failed or not working due to a change, and they must wait for a new illumination, etc.

Endpoint or (immediate) Accessibility: The likelihood that an arrangement will be functioning at any given randomized moment t is known as pinpoint or simultaneous uptime. It is represented as A(t) and is determined by the system's predicted durability.

Estimated Reliability of Downtime: The percentage of an assignment or interval of time whereby the entire system is usable is known as the mean accessibility. It shows the current accessible function's average across the time interval (0, T). Am(T) is used to represent it.

$$\text{So: } A(T) = \frac{1}{1}\int_0^T A(t)\,dt$$

Possibility of a stable condition: The maximum value of the simultaneous reliability functional that gradually gets closer to infinitely is the system's perpetual accessibility.

$$\text{So: } A\ (\infty) = \lim_{t\to\infty} A(t)$$

Reliability of Operations: An assessment of reliability that takes into account every identified cause of unavailability is called operating reliability. The functional dependability formula is:

$$A_0 = \frac{Uptime}{Operational\ Cycle},$$

Where in availability is the total amount of time the equipment was operationally throughout the course of its operations, and operations cycles is the total amount of time the technology was in operations under investigation. Functional accessibility, then, is the accessibility that clients truly encounter. In essence, it is the post-event unavailability dependent on real-world systems occurrences. Next, the temporary condition is modeled mathematically: The mathematical models controlling an industrial plant's accessibility are the ones derived employing the change in figure 1 as a starting point: Temporary situation in which the incidences of malfunction and maintenance fluctuate:

$$\frac{d\rho_1(t)}{dt} + [\phi_1(\omega) + \phi_2(\omega) + \phi_3(\omega) + \phi_4(\omega) + \phi_5(\omega) + \phi_6(\omega) + \phi_7(\omega) + \phi_8(\omega) + \lambda_1]\rho_1(t) = \int \big[\eta_8(v)\rho_1(v,\omega,t)\,d\omega\,d\omega + \int \eta_1(v)\rho_2(v,\omega,t) + \int \eta_2(v)\rho_3(v,\omega,t) + \int \eta_3(v)\rho_4(v,\omega,t) + \int \eta_4(v)\rho_5(v,\omega,t) + \int \eta_5(v)\rho_6(v,\omega,t) + \int \eta_6(v)\rho_7(v,\omega,t) + \int \eta_7(v)\rho_8(v,\omega,t)\big]\,d\omega\,d\omega + \int \psi_1(v)\rho_9(v,t)\mathrm{dv} \tag{1}$$

$$\frac{\partial\rho_2(v,\omega,t)}{\partial t} + \frac{\partial\rho_2(v,\omega,t)}{\partial v} + \frac{\partial\rho_2(v,\omega,t)}{\partial w}\phi_8(w)\rho_2(v,w,t) + \eta_1(v)\rho_2(v,w,t) = \phi_1(w)\rho_1(t) + \eta_8(v)\rho_{18}(v,w,t) + \psi_1(v)\rho_{10}(v,\omega,t) \tag{2}$$

$$\frac{\partial\rho_3(v,\omega,t)}{\partial t} + \frac{\partial\rho_3(v,\omega,t)}{\partial v} + \frac{\partial\rho_3(v,\omega,t)}{\partial w}\phi_8(w)\rho_3(v,w,t) + \eta_2(v)\rho_3(v,w,t) = \phi_2(w)\rho_1(t) + \eta_8(v)\rho_{19}(v,w,t) + \psi_1(v)\rho_{11}(v,\omega,t) \tag{3}$$

$$\frac{\partial\rho_4(v,\omega,t)}{\partial t} + \frac{\partial\rho_4(v,\omega,t)}{\partial v} + \frac{\partial\rho_4(v,\omega,t)}{\partial w}\phi_8(w)\rho_4(v,w,t) + \eta_3(v)\rho_4(v,w,t) = \phi_3(w)\rho_1(t) + \eta_8(v)\rho_{20}(v,w,t) + \psi_1(v)\rho_{12}(v,\omega,t) \tag{4}$$

$$\frac{\partial\rho_5(v,\omega,t)}{\partial t} + \frac{\partial\rho_5(v,\omega,t)}{\partial v} + \frac{\partial\rho_5(v,\omega,t)}{\partial w}\phi_8(w)\rho_5(v,w,t) + \eta_4(v)\rho_5(v,w,t) = \phi_4(w)\rho_1(t) + \eta_8(v)\rho_{21}(v,w,t) + \psi_1(v)\rho_{13}(v,\omega,t) \tag{5}$$

$$\frac{\partial\rho_6(v,\omega,t)}{\partial t} + \frac{\partial\rho_6(v,\omega,t)}{\partial v} + \frac{\partial\rho_6(v,\omega,t)}{\partial w}\phi_8(w)\rho_6(v,w,t) + \eta_5(v)\rho_6(v,w,t) = \phi_5(w)\rho_1(t) + \eta_8(v)\rho_{22}(v,w,t) + \psi_1(v)\rho_{14}(v,\omega,t) \tag{6}$$

$$\frac{\partial\rho_7(v,\omega,t)}{\partial t} + \frac{\partial\rho_7(v,\omega,t)}{\partial v} + \frac{\partial\rho_7(v,\omega,t)}{\partial w}\phi_8(w)\rho_7(v,w,t) + \eta_6(v)\rho_7(v,w,t) = \phi_6(w)\rho_1(t) + \eta_8(v)\rho_{23}(v,w,t) + \psi_1(v)\rho_{15}(v,\omega,t) \tag{7}$$

$$\frac{\partial\rho_8(v,\omega,t)}{\partial t} + \frac{\partial\rho_8(v,\omega,t)}{\partial v} + \frac{\partial\rho_8(v,\omega,t)}{\partial w} + \phi_1(w)\rho_8(v,w,t) + \phi_8(w)\rho_8(v,w,t) + \eta_7(v)\rho_8(v,w,t) = \phi_7(w)\rho_1(t) +$$

$$\eta_1(v)\rho_{25}(v,w,t)+\eta_8(v)\rho_{24}(v,w,t)+\psi_1(v)\rho_{16}(v,\omega,t) \tag{8}$$

$$\frac{\partial\rho_9(v,\omega,t)}{\partial t}+\frac{\partial\rho_9(v,\omega,t)}{\partial v}+\frac{\partial\rho_9(v,\omega,t)}{\partial w}+\phi_1(w)\rho_9(v,w,t)+\phi_2(w)\rho_9(v,w,t)+\phi_3(w)\rho_9(v,w,t)+\phi_4(w)\rho_9(v,w,t)+\phi_5(w)\rho_9(v,w,t)+\phi_6(w)\rho_9(v,w,t)+\phi_7(w)\rho_9(v,w,t)+\eta_7(v)\rho_8(v,w,t)+\psi_1(v)\rho_9(v,\omega,t)=\lambda_1\rho_1(t)+\eta_1(v)\rho_{10}(v,w,t)+\eta_2(v)\rho_{11}(v,w,t)+\eta_3(v)\rho_{12}(v,w,t)+\eta_4(v)\rho_{13}(v,w,t)+\eta_5(v)\rho_{14}(v,w,t)+\eta_6(v)\rho_{15}(v,w,t)+\eta_7(v)\rho_{16}(v,w,t) \tag{9}$$

$$\frac{\partial\rho_{10}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{10}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{10}(v,\omega,t)}{\partial w}+\eta_1(v)\rho_{10}(v,w,t)+\psi_1(v)\rho_{10}(v,w,t)=\phi_1(w)\rho_9(v,w,t) \tag{10}$$

$$\frac{\partial\rho_{11}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{11}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{11}(v,\omega,t)}{\partial w}+\eta_2(v)\rho_{11}(v,w,t)+\psi_1(v)\rho_{11}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{11}$$

$$\frac{\partial\rho_{12}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{12}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{12}(v,\omega,t)}{\partial w}+\eta_3(v)\rho_{12}(v,w,t)+\psi_1(v)\rho_{12}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{12}$$

$$\frac{\partial\rho_{13}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{13}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{13}(v,\omega,t)}{\partial w}+\eta_4(v)\rho_{13}(v,w,t)+\psi_1(v)\rho_{13}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{13}$$

$$\frac{\partial\rho_{14}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{14}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{14}(v,\omega,t)}{\partial w}+\eta_5(v)\rho_{14}(v,w,t)+\psi_1(v)\rho_{14}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{14}$$

$$\frac{\partial\rho_{15}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{15}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{15}(v,\omega,t)}{\partial w}+\eta_6(v)\rho_{15}(v,w,t)+\psi_1(v)\rho_{15}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{15}$$

$$\frac{\partial\rho_{16}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{16}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{16}(v,\omega,t)}{\partial w}+\eta_7(v)\rho_{16}(v,w,t)+\psi_1(v)\rho_{16}(v,w,t)=\phi_1(w)\rho_9(v,t) \tag{16}$$

$$\frac{\partial\rho_{17}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{17}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{17}(v,\omega,t)}{\partial w}+\eta_8(v)\rho_{17}(v,w,t)=\phi_8(w)\rho_1(t)+\eta_1(v)\rho_{18}(v,w,t)+\eta_2(v)\rho_{19}(v,w,t)+\eta_3(v)\rho_{20}(v,w,t)+\eta_4(v)\rho_{21}(v,w,t)+\eta_5(v)\rho_{22}(v,w,t)+\eta_6(v)\rho_{23}(v,w,t)+\eta_7(v)\rho_{24}(v,w,t) \tag{17}$$

$$\frac{\partial\rho_{18}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{18}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{18}(v,\omega,t)}{\partial w}+\eta_1(v)\rho_{18}(v,w,t)+\eta_8(v)\rho_{18}(v,w,t)=\phi_8(w)\rho_2(v,w,t) \tag{18}$$

$$\frac{\partial\rho_{19}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{19}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{19}(v,\omega,t)}{\partial w}+\eta_2(v)\rho_{19}(v,w,t)+\eta_8(v)\rho_{19}(v,w,t)=\phi_8(w)\rho_3(v,w,t) \tag{19}$$

$$\frac{\partial\rho_{20}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{20}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{20}(v,\omega,t)}{\partial w}+\eta_3(v)\rho_{20}(v,w,t)+\eta_8(v)\rho_{20}(v,w,t)=\phi_8(w)\rho_4(v,w,t) \tag{20}$$

$$\frac{\partial\rho_{21}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{21}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{21}(v,\omega,t)}{\partial w}+\eta_4(v)\rho_{21}(v,w,t)+\eta_8(v)\rho_{21}(v,w,t)=\phi_8(w)\rho_5(v,w,t) \tag{21}$$

$$\frac{\partial\rho_{22}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{22}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{22}(v,\omega,t)}{\partial w}+\eta_5(v)\rho_{22}(v,w,t)+\eta_8(v)\rho_{22}(v,w,t)=\phi_8(w)\rho_6(v,w,t) \tag{22}$$

$$\frac{\partial\rho_{23}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{23}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{23}(v,\omega,t)}{\partial w}+\eta_6(v)\rho_{23}(v,w,t)+\eta_8(v)\rho_{23}(v,w,t)=\phi_8(w)\rho_7(v,w,t) \tag{23}$$

$$\frac{\partial\rho_{24}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{24}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{24}(v,\omega,t)}{\partial w}+\eta_7(v)\rho_{24}(v,w,t)+\eta_8(v)\rho_{24}(v,w,t)=\phi_8(w)\rho_{24}(v,w,t) \tag{24}$$

$$\frac{\partial\rho_{25}(v,\omega,t)}{\partial t}+\frac{\partial\rho_{25}(v,\omega,t)}{\partial v}+\frac{\partial\rho_{25}(v,\omega,t)}{\partial w}+\eta_1(v)\rho_{25}(v,w,t)=\phi_1(w)\rho_8(v,w,t) \tag{25}$$

Prerequisites of Boundaries:

The following are the subsystem's boundaries:

$$\rho_2(0,w,t)=\phi_1(\omega)\rho_1(t) \tag{26}$$

$$\rho_3(0,w,t)=\phi_2(\omega)\rho_1(t) \tag{27}$$

$$\rho_4(0,w,t)=\phi_3(\omega)\rho_1(t) \tag{28}$$

$$\rho_5(0,w,t)=\phi_4(\omega)\rho_1(t) \tag{29}$$

$$\rho_6(0,w,t)=\phi_5(\omega)\rho_1(t) \tag{30}$$

$$\rho_7(0,w,t)=\phi_6(\omega)\rho_1(t) \tag{31}$$

$$\rho_8(0,w,t)=\phi_7(\omega)\rho_1(t) \tag{32}$$

$$\rho_9(0,w,t) = \lambda_1\rho_1(t) \tag{33}$$

$$\rho_{10}(0,w,t) = \int \phi_1(\omega)\rho_9(v,t)dv \tag{34}$$

$$\rho_{11}(0,w,t) = \int \phi_2(\omega)\rho_9(v,t)dv \tag{35}$$

$$\rho_{12}(0,w,t) = \int \phi_3(\omega)\rho_9(v,t)dv \tag{36}$$

$$\rho_{13}(0,w,t) = \int \phi_4(\omega)\rho_9(v,t)dv \tag{37}$$

$$\rho_{14}(0,w,t) = \int \phi_5(\omega)\rho_9(v,t)dv \tag{38}$$

$$\rho_{15}(0,w,t) = \int \phi_6(\omega)\rho_9(v,t)dv \tag{39}$$

$$\rho_{16}(0,w,t) = \int \phi_7(\omega)\rho_9(v,t)dv \tag{40}$$

$$\rho_{17}(0,w,t) = \phi_8(\omega)\rho_1(t) \tag{41}$$

$$\rho_{18}(0,w,t) = \int \phi_8(\omega)\rho_2(v,w,t)dv \tag{42}$$

$$\rho_{19}(0,w,t) = \int \phi_8(\omega)\rho_3(v,w,t)dv \tag{43}$$

$$\rho_{20}(0,w,t) = \int \phi_8(\omega)\rho_4(v,w,t)dv \tag{44}$$

$$\rho_{21}(0,w,t) = \int \phi_8(\omega)\rho_5(v,w,t)dv \tag{45}$$

$$\rho_{22}(0,w,t) = \int \phi_8(\omega)\rho_6(v,w,t)dv \tag{46}$$

$$\rho_{23}(0,w,t) = \int \phi_8(\omega)\rho_7(v,w,t)dv \tag{47}$$

$$\rho_{24}(0,w,t) = \int \phi_8(\omega)\rho_8(v,w,t)dv \tag{48}$$

$$\rho_{25}(0,w,t) = \int \phi_1(\omega)\rho_8(v,w,t)dv \tag{49}$$

Starting circumstances: The sub-system's starting point is as follows:

$$\rho_1(0) = 1 \tag{50}$$

$$\rho_9(v,0) = 0 \tag{51}$$

$$\rho_2(v,w,0) = 0 \tag{52}$$

$$\rho_3(v,w,0) = 0$$

$$\rho_4(v,w,0) = 0$$

$$\rho_5(v,w,0) = 0$$

$$\rho_6(v,w,0) = 0$$

$$\rho_7(v,w,0) = 0$$

$$\rho_8(v,w,0) = 0$$

$$\rho_9(v,w,0) = 0$$

$$\rho_{10}(v,w,0) = 0$$

$$\rho_{11}(v,w,0) = 0$$

$$\rho_{12}(v,w,0) = 0$$

$$\rho_{13}(v,w,0) = 0$$

$$\rho_{14}(v,w,0) = 0$$

$$\rho_{15}(v,w,0) = 0$$

$$\rho_{16}(v,w,0) = 0$$

$$\rho_{17}(v,w,0) = 0$$

$$\rho_{18}(v,w,0)=0$$

$$\rho_{19}(v,w,0)=0$$

$$\rho_{20}(v,w,0)=0$$

$$\rho_{21}(v,w,0)=0$$

$$\rho_{22}(v,w,0)=0$$

$$\rho_{23}(v,w,0)=0$$

$$\rho_{24}(v,w,0)=0$$

$$\rho_{25}(v,w,0)=0$$

Chapman - Kolmogorov differential distinction at the same time is a collection of various equations (1) to (25) plus the boundary conditions (26) to (49) and beginning constraint (50) to (52). While solutions (2) through (25) are linear partial differential formulas, problem (1) is a first order nonlinear differential model.

The following lists the likelihoods that were thus determined regarding every condition:

$$\rho_1(t)= e^{-[\phi_1(\omega)+\phi_2(\omega)+\phi_3(\omega)+\phi_4(\omega)+\phi_5(\omega)+\phi_6(\omega)+\phi_7(\omega)+\phi_8(\omega)+\lambda_1 t]} + \int \Big\{ \int \eta_8(v)\rho_{17}(v,w,t)dv\,dw + \Big[\int \eta_1(v)\rho_2(v,w,t)+\eta_2(v)\rho_3(v,w,t)+\eta_3(v)\rho_4(v,w,t)+\eta_4(v)\rho_5(v,w,t)+\eta_5(v)\rho_6(v,w,t)+\eta_6(v)\rho_7(v,w,t)+\eta_7(v)\rho_8(v,w,t)]dv\,dw + \int \psi_1(v)\rho_9(v,t)dv\Big].e^{[\phi_1(\omega)t+\phi_2(\omega)t+\phi_3(\omega)t+\phi_4(\omega)t+\phi_5(\omega)t+\phi_6(\omega)t+\phi_7(\omega)t+\lambda_1 t]}\Big\}\,dt \tag{53}$$

$$\rho_2(v,w,t)=e^{-\int[\phi_8(w)+\eta_1(v)]dv}\Big[\phi_1(w-v)\rho_1(t-v)+\int\{\phi_1(w)\rho_1(t)+\eta_8(v)\rho_{18}(v,\omega,t)+\psi_1(v)\rho_{10}(v,w,t)\}.e^{\int[\phi_8(w)\cdot+\eta_1(v)]dv}dv\Big] \tag{54}$$

$$\rho_3(v,w,t)=e^{-\int[\phi_8(w)\cdot+\eta_2(v)]dv}\Big[\phi_2(w-v)\rho_2(t-v)+\int\{\phi_2(w)\rho_1(t)+\eta_8(v)\rho_{19}(v,\omega,t)+\psi_1(v)\rho_{11}(v,w,t)\}.e^{\int[\phi_8(w)\cdot+\eta_3(v)]dv}\Big]dv \tag{55}$$

$$\rho_4(v,w,t)=e^{-\int[\phi_8(w)\cdot+\eta_3(v)]dv}\Big[\phi_3(w-v)\rho_3(t-v)+\int\{\phi_3(w)\rho_1(t)+\eta_8(v)\rho_{20}(v,\omega,t)+\psi_1(v)\rho_{12}(v,w,t)\}.e^{\int[\phi_8(w)\cdot+\eta_3(v)]dv}\Big]dv \tag{56}$$

$$\rho_5(v,w,t)=e^{-\int[\phi_8(w)\cdot+\eta_4(v)]dv}\Big[\phi_4(w-v)\rho_4(t-v)+\int\{\phi_4(w)\rho_1(t)+\eta_8(v)\rho_{21}(v,\omega,t)+\psi_1(v)\rho_{13}(v,w,t)\}.e^{\int[\phi_8(w)+\eta_4(v)]dv}dv\Big] \tag{57}$$

$$\rho_6(v,w,t)=e^{-\int[\phi_8(w)\cdot+\eta_5(v)]dv}\Big[\phi_5(w-v)\rho_5(t-v)+\int\{\phi_5(w)\rho_1(t)+\eta_8(v)\rho_{22}(v,\omega,t)+\psi_1(v)\rho_{14}(v,w,t)\}.e^{\int[\phi_8(w)+\eta_5(v)]dv}dv\Big] \tag{58}$$

$$\rho_7(v,w,t)=e^{-\int[\phi_8(w)\cdot+\eta_6(v)]dv}\Big[\phi_6(w-v)\rho_6(t-v)+\int\{\phi_6(w)\rho_1(t)+\eta_8(v)\rho_{22}(v,\omega,t)+\psi_1(v)\rho_{15}(v,w,t)\}.e^{\int[\phi_8(w)+\eta_6(v)]dv}dv\Big] \tag{59}$$

$$\rho_8(v,w,t)=e^{-\int[\phi_1(w)+\phi_8(w)+\eta_7(v)]dv}\Big[\phi7(w-v)\rho_7(t-v)+\int\{\phi_7(w)\rho_1(t)+\eta_1(v)\rho_{25}(v,\omega,t)+\eta_8(v)\rho_{24}(v,\omega,t)+\psi_1(v)\rho_{16}(v,w,t)\}.e^{\int[\phi_1(w)+\phi_8(w)+\eta_7(v)]dv}dv\Big] \tag{60}$$

$$\rho_9(v,t)=e^{-\int[\phi_1(w)+\Psi1(v)+\phi2(w)+\Psi2(v)+\phi3(w)+\Psi3(v)+\phi4(w)+\Psi4(v)+\phi5(w)+\Psi5(v)+\phi6(w)+\Psi6(v)+\phi7(w)+\Psi7(v)]dv} * \Big[\lambda_1\rho_1(t-v)+\int\{\lambda_1\rho_1(t)+\eta_1(v)\rho_{10}(v,\omega,t)+\eta_2(v)\rho_{11}(v,\omega,t)+\eta_3(v)\rho_{12}(v,\omega,t)+\eta_4(v)\rho_{13}(v,\omega,t)+\eta_5(v)\rho_{14}(v,\omega,t)+\eta_6(v)\rho_{15}(v,\omega,t)+\eta_7(v)\rho_{16}(v,\omega,t)\} * e^{\int\phi_1(w)+\phi_2(w)+\phi_3(w)+\phi_4(w)+\phi_5(w)+\phi_6(w)+\phi_7(w)+\psi_1(v)+\psi_2(v)+\psi_3(v)+\psi_4(v)+\psi_5(v)+\psi_6(v)+\psi_7(v)}\Big]dv \tag{61}$$

$$\rho_{10}(0,w,t)=e^{-\int[\eta_1(v)+\Psi1(v)]dv} * \Big[\int\phi_1(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_1(v)+\Psi1(v)]dv}+\int\phi_1(w-v)\rho_9(v,t-v)\ dv\Big] \tag{62}$$

$$\rho_{11}(v,w,t)=e^{-\int[\eta_2(v)+\Psi1(v)]dv} * \Big[\int\phi_1(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_2(v)+\Psi1(v)]dv}+\int\phi_1(w-v)\rho_9(v,t-v)\ dv\Big] \tag{63}$$

$$\rho_{12}(v,w,t)=e^{-\int[\eta_3(v)+\Psi1(v)]dv}.\Big[\int\big[\phi_1(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_3(v)+\Psi1(v)]d\omega}+\int\phi_1(w-v)\rho_9(v,t-v)\big]dv\Big] \tag{64}$$

$$\rho_{13}(v,w,t) = e^{-\int[\eta_4(v)+\Psi 1(v)]\,dv}.\left[\int \phi_1\,(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_4(v)+\Psi 1(v)]\,d\omega} + \int \phi_1\,(w-v)\rho_9(v,t-v)\;dv\right] \tag{65}$$

$$\rho_{14}(v,w,t) = e^{-\int[\eta_5(v)+\Psi 1(v)]\,d\omega}.\left[\int \phi_1\,(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_5(v)+\Psi 1(v)]\,d\omega} + \int \phi_1\,(w-v)\rho_9(v,t-v)\;dv\right] \tag{66}$$

$$\rho_{15}(v,w,t) = e^{-\int[\eta_6(v)+\Psi 1(v)]\,d\omega}.\left[\int \phi_1\,(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_6(v)+\Psi 1(v)]\,d\omega} + \int \phi_1\,(w-v)\rho_9(v,t-v)\;dv\right] \tag{67}$$

$$\rho_{16}(v,w,t) = e^{-\int[\eta_7(v)+\Psi 1(v)]\,d\omega}.\left[\int \phi_1\,(w-v)\rho_9(v,t-v)\,.\,e^{\int[\eta_7(v)+\Psi 1(v)]\,d\omega} + \int \phi_1\,(w-v)\rho_9(v,t-v)\;dv\right] \tag{68}$$

$$\begin{gathered}\rho_{17}(v,w,t) = e^{-\int[\eta_8(v)dw+\Psi 1(v)]\,d\omega}.\Big[\int \phi_8\,(w-v)\rho_1(t-v) + \int\{\phi_8(w)\rho_1(t) + \eta_1(v)\rho_{18}(v,\omega,t) + \\ \eta_2(v)\rho_{19}(v,\omega,t) + \eta_3(v)\rho_{20}(v,\omega,t) + \eta_4(v)\rho_{21}(v,\omega,t) + \eta_5(v)\rho_{22}(v,\omega,t) + \eta_6(v)\rho_{23}(v,\omega,t) + \\ \eta_6(v)\rho_{24}(v,\omega,t)\} + e^{-\int[\eta_8(v)dw]\,d\omega}\;dv\Big]\end{gathered} \tag{69}$$

$$\rho_{18}(v,w,t) = e^{-\int[\eta_1(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_2(v,w,t).\,e^{\int[\eta_1(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_2(v,w-v,t-v)\;\;dv\right] \tag{70}$$

$$\rho_{19}(v,w,t) = e^{-\int[\eta_2(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_3(v,w,t).\,e^{\int[\eta_2(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_3(v,w-v,t-v)\;\;dv\right] \tag{71}$$

$$\rho_{20}(v,w,t) = e^{-\int[\eta_3(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_4(v,w,t).\,e^{\int[\eta_3(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_4(v,w-v,t-v)\;\;dv\right] \tag{72}$$

$$\rho_{21}(v,w,t) = e^{-\int[\eta_4(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_5(v,w,t).\,e^{\int[\eta_4(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_5(v,w-v,t-v)\;\;dv\right] \tag{73}$$

$$\rho_{22}(v,w,t) = e^{-\int[\eta_5(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_6(v,w,t).\,e^{\int[\eta_5(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_6(v,w-v,t-v)\;\;dv\right] \tag{74}$$

$$\rho_{23}(v,w,t) = e^{-\int[\eta_6(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_6(v,w,t).\,e^{\int[\eta_6(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_7(v,w-v,t-v)\;\;dv\right] \tag{75}$$

$$\rho_{24}(v,w,t) = e^{-\int[\eta_7(v)+\eta_8(v)]\,d\omega}.\left[\int \phi_8\,(w)\rho_7(v,w,t).\,e^{\int[\eta_7(v)+\eta_8(v)]\,d\omega} + \int \phi_8\,(w-v)\rho_8(v,w-v,t-v)\;\;dv\right] \tag{76}$$

$$\rho_{25}(v,w,t) = e^{-\int[\eta_1(v)]\,d\omega}.\left[\int \phi_1\,(w)\rho_8(v,w,t).\,e^{\int[\eta_1(v)]\,d\omega} + \int \phi_1\,(w-v)\rho_8(v,w-v,t-v)\;\;dv\right] \tag{77}$$

Therefore, the system's time invariant unavailability A(t) is provided by

$$\begin{gathered}\mathrm{A(t)} = \rho_1(t) \\ = e^{[\phi_1(\omega)t+\phi_2(\omega)t+\phi_3(\omega)t+\phi_4(\omega)t+\phi_5(\omega)t+\phi_6(\omega)t+\phi_7(\omega)t+\phi_8(\omega)t+\lambda_1 t]}\Big\{1 + \int\Big[\{\eta_8(v)\rho_{17}(v,\omega,t)\,dv\,dw\} + \\ \int \eta_1\,(v)\rho_2(v,\omega,t) + \int \eta_2\,(v)\rho_3(v,\omega,t) + \int \eta_3\,(v)\rho_4(v,\omega,t) + \int \eta_4\,(v)\rho_5(v,\omega,t) + \int \eta_5\,(v)\rho_6(v,\omega,t) + \\ \int \eta_6\,(v)\rho_7(v,\omega,t) + \int \eta_7\,(v)\rho_8(v,\omega,t)\Big]dw\,dw\; + \\ \Psi 1(v)\rho_9(v,t)\,dv\Big\}.\,e^{[\phi_1(\omega)t+\phi_2(\omega)t+\phi_3(\omega)t+\phi_4(\omega)t+\phi_5(\omega)t+\phi_6(\omega)t+\phi_7(\omega)t+\phi_8(\omega)t+\lambda_1 t]}dt\end{gathered} \tag{78}$$

Temporary conditions in which the incidences of breakdown and regeneration are equally unchanged: -

The collection of equations (1) to (25) can be condensed to the following form for the purpose of determining the network's unavailability, assuming equal breakdown and restoration frequencies are unchanged:

$$\begin{gathered}\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \\ \eta_1(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t)\end{gathered} \tag{79}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) =$$

$$\eta_2(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{80}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \eta_3(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{81}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \eta_4(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{82}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \eta_5(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{83}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \eta_6(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{84}$$

$$\frac{d\rho_1(t)}{dt} + \phi_1\rho_1(t) + \phi_2\rho_1(t) + \phi_3\rho_1(t) + \phi_4\rho_1(t) + \phi_5\rho_1(t) + \phi_6\rho_1(t) + \phi_7\rho_1(t) + \phi_8\rho_1(t) + \lambda_1\rho_1(t) = \eta_7(v)\rho_2(t) + \psi_1\rho_9(t) + \eta_8\rho_{17}(t) \tag{85}$$

$$\frac{d\rho_2(t)}{dt} + \phi_8\rho_2(t) + \eta_1\rho_2(t) = \phi_1\rho_1(t) + \eta_8\rho_{18}(t) + \psi_1\rho_{10}(t) \tag{86}$$

$$\frac{d\rho_3(t)}{dt} + \phi_8\rho_3(t) + \eta_2\rho_3(t) = \phi_2\rho_1(t) + \eta_8\rho_{19}(t) + \psi_1\rho_{11}(t) \tag{87}$$

$$\frac{d\rho_4(t)}{dt} + \phi_8\rho_4(t) + \eta_3\rho_4(t) = \phi_3\rho_1(t) + \eta_8\rho_{20}(t) + \psi_1\rho_{12}(t) \tag{88}$$

$$\frac{d\rho_5(t)}{dt} + \phi_8\rho_5(t) + \eta_4\rho_5(t) = \phi_4\rho_1(t) + \eta_8\rho_{21}(t) + \psi_1\rho_{13}(t) \tag{89}$$

$$\frac{d\rho_6(t)}{dt} + \phi_8\rho_6(t) + \eta_5\rho_6(t) = \phi_5\rho_1(t) + \eta_8\rho_{22}(t) + \psi_1\rho_{14}(t) \tag{90}$$

$$\frac{d\rho_7(t)}{dt} + \phi_8\rho_7(t) + \eta_6\rho_7(t) = \phi_6\rho_1(t) + \eta_8\rho_{23}(t) + \psi_1\rho_{15}(t) \tag{91}$$

$$\frac{d\rho_8(t)}{dt} + \phi_8\rho_8(t) + \eta_7\rho_8(t) = \phi_7\rho_1(t) + \eta_8\rho_{24}(t) + \psi_1\rho_{16}(t) \tag{92}$$

$$\frac{d\rho_9(t)}{dt} + \phi_1\rho_9(t) + \phi_2\rho_9(t) + \phi_3\rho_9(t) + \phi_4\rho_9(t) + \phi_5\rho_9(t) + \phi_6\rho_9(t) + \phi_7\rho_9(t) + \psi_1\rho_9(t) = \lambda_1\rho_1(t) + \eta_1\rho_{10}(t) + \eta_2\rho_{11}(t) + \eta_3\rho_{12}(t) + \eta_4\rho_{13}(t) + \eta_5\rho_{14}(t) + \eta_6\rho_{15}(t) + \eta_7\rho_{16}(t) \tag{93}$$

$$\frac{d\rho_{10}(t)}{dt} + \eta_1\rho_{10}(t) + \psi_1\rho_{10}(t) = \phi_1\rho_9(t) \tag{94}$$

$$\frac{d\rho_{11}(t)}{dt} + \eta_2\rho_{11}(t) + \psi_1\rho_{11}(t) = \phi_2\rho_9(t) \tag{95}$$

$$\frac{d\rho_{12}(t)}{dt} + \eta_3\rho_{12}(t) + \psi_1\rho_{12}(t) = \phi_3\rho_9(t) \tag{96}$$

$$\frac{d\rho_{13}(t)}{dt} + \eta_4\rho_{13}(t) + \psi_1\rho_{13}(t) = \phi_4\rho_9(t) \tag{97}$$

$$\frac{d\rho_{14}(t)}{dt} + \eta_5\rho_{14}(t) + \psi_1\rho_{14}(t) = \phi_5\rho_9(t) \tag{98}$$

$$\frac{d\rho_{15}(t)}{dt} + \eta_6\rho_{15}(t) + \psi_1\rho_{15}(t) = \phi_6\rho_9(t) \tag{99}$$

$$\frac{d\rho_{16}(t)}{dt} + \eta_7\rho_{16}(t) + \psi_1\rho_{16}(t) = \phi_7\rho_9(t) \tag{100}$$

$$\frac{d\rho_{17}(t)}{dt} + \eta_8\rho_{17}(t) = \phi_8\rho_1(t) + \eta_1\rho_{18}(t) + \eta_2\rho_{19}(t) + \eta_3\rho_{20}(t) + \eta_4\rho_{21}(t) + \eta_5\rho_{22}(t) + \eta_6\rho_{23}(t) + \eta_7\rho_{24}(t) \tag{101}$$

$$\frac{d\rho_{18}(t)}{dt} + \eta_1\rho_{18}(t) + \eta_8\rho_{18}(t) = \phi_8\rho_2(t) \tag{102}$$

$$\frac{d\rho_{19}(t)}{dt} + \eta_2\rho_{19}(t) + \eta_8\rho_{19}(t) = \phi_8\rho_3(t) \tag{103}$$

$$\frac{d\rho_{20}(t)}{dt}+\eta_3\rho_{20}(t)+\eta_8\rho_{20}(t)=\phi_8\rho_4(t) \tag{104}$$

$$\frac{d\rho_{21}(t)}{dt}+\eta_4\rho_{21}(t)+\eta_8\rho_{21}(t)=\phi_8\rho_5(t) \tag{105}$$

$$\frac{d\rho_{22}(t)}{dt}+\eta_5\rho_{22}(t)+\eta_8\rho_{22}(t)=\phi_8\rho_6(t) \tag{106}$$

$$\frac{d\rho_{23}(t)}{dt}+\eta_6\rho_{23}(t)+\eta_8\rho_{23}(t)=\phi_8\rho_7(t) \tag{107}$$

$$\frac{d\rho_{24}(t)}{dt}+\eta_7\rho_{24}(t)+\eta_8\rho_{24}(t)=\phi_8\rho_8(t) \tag{108}$$

$$\frac{d\rho_{25}(t)}{dt}+\eta_1\rho_{25}(t)=\phi_1\rho_8(t) \tag{109}$$

Initial conditions: -

$$\rho_r=\begin{cases}1, r=1\\ 0\ , otherwise\end{cases} \tag{110}$$

The majority of publications have solved Chapman and inverse energy cascade problems using the technique of matrix analysis and Fourier conversion. Nonetheless, we have used the method described by the authors of (Backman et al., 2019) to use solution (110) to solve the collection of simultaneous equations (79) to (109). Given that t = 0.005 is one day, a computation had been performed from t = 0 to t = 360 days.

Equation (78) provides for the accessibility of a steel fabrication facility given current conditions where the whole thing is operating at its full potential.

4. Characterization of States of Stability with Consistent Conversion Increases

Since transitional frequencies are consistent and the leadership is constantly concerned with durability by establishing d/dt → 0 at t→∞, equations (79) to (109) look like this:

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_1(v)\rho_2(t)+\psi_1\rho_9(t) \tag{111}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_2(v)\rho_3(t)+\psi_1\rho_9(t) \tag{112}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_3(v)\rho_4(t)+\psi_1\rho_9(t) \tag{113}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_4(v)\rho_5(t)+\psi_1\rho_9(t) \tag{114}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_5(v)\rho_6(t)+\psi_1\rho_9(t) \tag{115}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_6(v)\rho_7(t)+\psi_1\rho_9(t) \tag{116}$$

$$\phi_1\rho_1(t)+\phi_2\rho_1(t)+\phi_3\rho_1(t)+\phi_4\rho_1(t)+\phi_5\rho_1(t)+\phi_6\rho_1(t)+\phi_7\rho_1(t)+\phi_8\rho_1(t)+\lambda_1\rho_1(t)= \\ \eta_7(v)\rho_8(t)+\psi_1\rho_9(t) \tag{117}$$

$$\phi_8\rho_2(t)+\ \eta_1\rho_2=\phi_1\rho_1+\eta_8\rho_{18}+\psi_1\rho_{10} \tag{118}$$

$$\phi_8\rho_3(t)+\ \eta_2\rho_3=\phi_2\rho_1+\eta_8\rho_{19}+\psi_1\rho_{11} \tag{119}$$

$$\phi_8\rho_4(t)+\ \eta_3\rho_4=\phi_3\rho_1+\eta_8\rho_{19}+\psi_1\rho_{12} \tag{120}$$

$$\phi_8\rho_5(t)+\ \eta_4\rho_5=\phi_4\rho_1+\eta_8\rho_{20}+\psi_1\rho_{13} \tag{121}$$

$$\phi_8\rho_6(t)+\ \eta_5\rho_6=\phi_5\rho_1+\eta_8\rho_{21}+\psi_1\rho_{14} \tag{122}$$

$$\phi_8\rho_7(t)+\ \eta_6\rho_7=\phi_6\rho_1+\eta_8\rho_{22}+\psi_1\rho_{15} \tag{123}$$

$$\phi_1\rho_8(t)+\phi_8\rho_8(t)+\ \eta_7\rho_8=\phi_7\rho_1+\eta_1\rho_{25}+\eta_8\rho_{24}+\psi_1\rho_{16} \tag{124}$$

$$\phi_1\rho_9(t)+\phi_2\rho_9(t)+\phi_3\rho_9(t)+\phi_4\rho_9(t)+\phi_5\rho_9(t)+\phi_6\rho_9(t)+\phi_7\rho_9(t)+\psi_1\rho_9=\lambda_1\rho_1+\eta_1\rho_{10}(t)+\eta_2\rho_{11}(t)+\eta_3\rho_{12}(t)+\eta_4\rho_{13}(t)+\eta_5\rho_{14}(t)+\eta_6\rho_{15}(t)+\eta_7\rho_{16}(t) \tag{125}$$

$$\eta_1\rho_{10}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{126}$$

$$\eta_2\rho_{11}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{127}$$

$$\eta_3\rho_{12}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{128}$$

$$\eta_4\rho_{13}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{129}$$

$$\eta_5\rho_{14}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{130}$$

$$\eta_6\rho_{15}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{131}$$

$$\eta_7\rho_{16}+\psi_1\rho_{10}=\phi_1\rho_9 \tag{132}$$

$$\eta_8\rho_{17}=\phi_1\rho_9+\eta_1\rho_{18}+\eta_2\rho_{19}+\eta_3\rho_{20}+\eta_4\rho_{21}+\eta_5\rho_{22}+\eta_5\rho_{23}+\eta_6\rho_{24} \tag{133}$$

$$\eta_1\rho_{18}+\eta_8\rho_{18}=\phi_8\rho_2 \tag{134}$$

$$\eta_2\rho_{19}+\eta_8\rho_{19}=\phi_8\rho_3 \tag{135}$$

$$\eta_3\rho_{20}+\eta_8\rho_{20}=\phi_8\rho_4 \tag{136}$$

$$\eta_4\rho_{21}+\eta_8\rho_{21}=\phi_8\rho_5 \tag{137}$$

$$\eta_5\rho_{22}+\eta_8\rho_{22}=\phi_8\rho_6 \tag{138}$$

$$\eta_6\rho_{23}+\eta_8\rho_{23}=\phi_8\rho_7 \tag{139}$$

$$\eta_7\rho_{24}+\eta_8\rho_{24}=\phi_8\rho_8 \tag{140}$$

$$\eta_1\rho_{25}=\phi_1\rho_1 \tag{141}$$

Repetitively evaluating this problem yields,

$$\rho_2=Q_8\,\rho_1 \tag{142}$$

$$\rho_3=Q_7\,\rho_1 \tag{143}$$

$$\rho_4=Q_6\,\rho_1 \tag{144}$$

$$\rho_5=Q_5\,\rho_1 \tag{145}$$

$$\rho_6=Q_4\,\rho_1 \tag{146}$$

$$\rho_7=Q_3\,\rho_1 \tag{147}$$

$$\rho_8=Q_2\,\rho_1 \tag{148}$$

$$\rho_9=Q_1\,\rho_1 \tag{149}$$

$$\rho_{10}=Q_9\,\rho_1 \tag{150}$$

$$\rho_{11}=Q_{10}\,\rho_1 \tag{151}$$

$$\rho_{12}=Q_{11}\rho_1 \tag{152}$$

$$\rho_{13}=Q_{12}\,\rho_1 \tag{153}$$

$$\rho_{14}=Q_{13}\,\rho_1 \tag{154}$$

$$\rho_{15}=Q_{14}\,\rho_1 \tag{155}$$

$$\rho_{16}=Q_{15}\,\rho_1 \tag{156}$$

$$\rho_{17}=Q_{16}\,\rho_1 \tag{157}$$

$$\rho_{18}=Q_{17}\,\rho_1 \tag{158}$$

$$\rho_{19}=Q_{18}\,\rho_1 \tag{159}$$

$$\rho_{20} = Q_{19}\,\rho_1 \tag{160}$$

$$\rho_{21} = Q_{20}\,\rho_1 \tag{161}$$

$$\rho_{22} = Q_{21}\,\rho_1 \tag{162}$$

$$\rho_{23} = Q_{22}\,\rho_1 \tag{163}$$

$$\rho_{24} = Q_{23}\,\rho_1 \tag{164}$$

$$\rho_{25} = Q_{24}\,\rho_1 \tag{165}$$

Had been

$$Q_1 = \frac{\lambda_1}{c_{15}c_{16}} \tag{166}$$

$$Q_2 = \frac{\frac{\phi_7}{c_{16}} + \frac{\psi_1\phi_7}{c_8 c_{16}}Q_1}{1 - \frac{\phi_1}{c_{16}} - \frac{\phi_8\eta_8}{\eta_1 c_{16}}} \tag{167}$$

$$Q_3 = \frac{\frac{\phi_6}{c_{17}} + \frac{\psi_1\phi_6}{c_9 c_{17}}Q_1}{1 - \frac{\phi_8\eta_8}{c_2 c_{17}}} \tag{168}$$

$$Q_4 = \frac{\frac{\phi_5}{c_{18}} + \frac{\psi_1\phi_5}{c_{10} c_{18}}Q_1}{1 - \frac{\phi_8\eta_8}{c_8 c_{18}}} \tag{169}$$

$$Q_5 = \frac{\frac{\phi_4}{c_{19}} + \frac{\psi_1\phi_5}{c_{19} c_{11}}Q_1}{1 - \frac{\phi_8\eta_8}{c_4 c_{19}}} \tag{170}$$

$$Q_6 = \frac{\frac{\phi_3}{c_{20}} + \frac{\psi_1\phi_8}{c_{12} c_{20}}Q_1}{1 - \frac{\phi_8\eta_8}{c_5 c_{20}}} \tag{171}$$

$$Q_7 = \frac{\frac{\phi_2}{c_{21}} + \frac{\psi_1\phi_2}{c_{11} c_{21}}Q_1}{1 - \frac{\phi_8\eta_8}{c_6 c_{21}}} \tag{172}$$

$$Q_8 = \frac{\frac{\phi_1}{c_{22}} + \frac{\psi_1\phi_1}{c_{14} c_{22}}Q_1}{1 - \frac{\phi_8\eta_8}{c_7 c_{22}}} \tag{173}$$

$$Q_9 = \frac{\phi_1}{c_{14}}Q_1 \tag{174}$$

$$Q_{10} = \frac{\phi_2}{c_{13}}Q_1 \tag{175}$$

$$Q_{11} = \frac{\phi_3}{c_{12}}Q_1 \tag{176}$$

$$Q_{12} = \frac{\phi_4}{c_{11}}Q_1 \tag{177}$$

$$Q_{13} = \frac{\phi_5}{c_{10}}Q_1 \tag{178}$$

$$Q_{14} = \frac{\phi_6}{c_9}Q_1 \tag{179}$$

$$Q_{15} = \frac{\phi_7}{c_8}Q_1 \tag{180}$$

$$Q_{16} = \frac{\phi_8\rho_1}{\eta_8} + \eta_1\frac{\phi_8}{C_7}\rho_2 + \eta_2\frac{\phi_8}{C_6}\rho_3 + \eta_3\frac{\phi_8}{C_5}\rho_4 + \eta_4\frac{\phi_8}{C_4}\rho_5 + \eta_5\frac{\phi_8}{C_3}\rho_6 + \eta_6\frac{\phi_8}{C_2}\rho_7 + \eta_7\frac{\phi_8}{C_1}\rho_8 \tag{181}$$

$$Q_{17} = \frac{\phi_8}{c_7}Q_8 \tag{182}$$

$$Q_{18} = \frac{\phi_8}{c_6}Q_6 \tag{183}$$

$$Q_{19} = \frac{\phi_8}{c_5} Q_7 \tag{184}$$

$$Q_{20} = \frac{\phi_8}{c_4} Q_5 \tag{185}$$

$$Q_{21} = \frac{\phi_8}{c_3} Q_4 \tag{186}$$

$$Q_{22} = \frac{\phi_8}{c_2} Q_4 \tag{187}$$

$$Q_{23} = \frac{\phi_9}{c_1} Q_2 \tag{188}$$

$$Q_{24} = 1 - \left[\frac{\eta_1\phi_1}{c_{14}} + \frac{\eta_2\phi_2}{c_{13}} + \frac{\eta_3\phi_3}{c_{12}} + \frac{\eta_4\phi_4}{c_{11}} + \frac{\eta_5\phi_5}{c_{10}} + \frac{\eta_6\phi_6}{c_9} + \frac{\eta_7\phi_7}{c_8}\right] * \frac{1}{c_{15}} \tag{189}$$

$$c_7 = \eta_1 + \eta_8 \tag{190}$$

$$c_6 = \eta_2 + \eta_8 \tag{191}$$

$$c_5 = \eta_3 + \eta_8 \tag{192}$$

$$c_4 = \eta_4 + \eta_8 \tag{193}$$

$$c_3 = \eta_5 + \eta_8 \tag{194}$$

$$c_2 = \eta_6 + \eta_8 \tag{195}$$

$$c_1 = \eta_7 + \eta_8 \tag{196}$$

$$c_7 = \eta_1 + \psi_1 \tag{197}$$

$$c_6 = \eta_2 + \psi_1 \tag{198}$$

$$c_5 = \eta_3 + \psi_1 \tag{199}$$

$$c_4 = \eta_4 + \psi_1 \tag{200}$$

$$c_3 = \eta_5 + \psi_1 \tag{201}$$

$$c_2 = \eta_6 + \psi_1 \tag{202}$$

$$c_1 = \eta_7 + \psi_1 \tag{203}$$

$$c_{15} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 + \phi_7 + \psi_1 \tag{204}$$

$$c_{16} = \phi_1 + \phi_8 + \eta_7 \tag{205}$$

$$c_{17} = \phi_8 + \eta_6 \tag{206}$$

$$c_{18} = \phi_8 + \eta_5 \tag{207}$$

$$c_{19} = \phi_8 + \eta_4 \tag{208}$$

$$c_{20} = \phi_8 + \eta_3 \tag{209}$$

$$c_{21} = \phi_8 + \eta_2 \tag{210}$$

$$c_{22} = \phi_8 + \eta_1 \tag{211}$$

Applying normalization assumptions immediately

$$\rho_1 + \rho_2 + \rho_3 +\cdot \cdots\cdots + \rho_{25} = 1 \tag{212}$$

If ρ_1 is established, researchers can ascertain

$$\rho_1, \rho_2, \rho_3, \cdot \cdots\cdots, \rho_{24}.$$

Lastly, the reliability of the system under current conditions, operating at its maximum potential, can be computed as

$$A_{\Sigma} = \rho_1 = \frac{1}{1+Q1+Q2+Q3+\ldots.+Q24} \tag{213}$$

5. Research of Personality

Utilizing the fourth-order Runge - Kutta method, the ensemble of linear equations (79) to (109), with starting point (110) was successfully solved computationally. Using conceivable combinations of the breakdown as well as maintenance rates, the likelihood that the technician will remain idle in both the transitory and stable states had been computed in table 1:

Table 1: Combination of Failure Rate and Repair Rate

Φ_1=0.002	η_1=0.09
Φ_2=0.003	η_2=0.03
Φ_3=0.003	η_3=0.04
Φ_4=0.005	η_4=0.07
Φ_5=0.002	η_5=0.09
Φ_6=0.004	η_6=0.02
Φ_7=0.003	η_7=0.06
Φ_8=0.01	η_8=0.062
λ_1=0.03	$\psi_1 = 0.02$

Once the framework is completely functional. Fig. 2 displays the calculated system capacity beneath current conditions.

5.1 Long-Term Accessibility

The following table displays the availability over the long term whenever the grinder machine's failing and scheduled upkeep speeds are varied, and all additional variables are fixed: Impact of grinder breakdowns (ϕ_8) and preventative upkeep frequencies on reliability.

Table 2: Impact of Failure and Preventative Maintenance Rates of Grinding Machines on Availability

$\lambda_1 \rightarrow \phi_8$	0.03	0.05	0.07	0.09	Transition Rates
0.01	0.9436	0.9298	0.9165	0.9036	Φ_1=0.002 Φ_2=0.003, Φ_3=0.003, Φ_4=0.005, Φ_5=0.002,
0.02	0.8710	0.8592	0.8477	0.8367	Φ_7=0.003, Φ_8=0.01 ψ_1=0.2,
0.03	0.8099	0.7995	0.7894	0.7797	η_1=0.09, η_2=0.03, η_3=0.04, η_4=0.07, η_5=0.09, η_6=0.02,
0.04	0.7616	0.7523	0.7533	0.7547	η_7=0.06, η_8=0.062,

Table 2 illustrates whether the milling equipment's breakdown and preventative upkeep frequencies affect the system's uptime. The findings show that a rise in pounding equipment breakdowns (ϕ_8) reduces longevity by 18.2% to 16.8% and the rate of preventative care by roughly 4.8% to 3.5%. This demonstrates that the rate of polishing equipment breakdowns has a greater impact on the aluminum producing industry's durability than the percentage of preventative upkeep. Impact of the cleaning and hot Steckel "M" machinery failure probability on long-term accessibility:

Table 3: Impact of Failure Rates of Descaling and Hot Steckel Mills on Long-Term Availability

$\Phi_1 \rightarrow \phi_6 \downarrow$	0.0002	0.0004	0.0006	0.0008	Transition Rates
0.0004	0.9436	0.9298	0.9265	0.9136	Φ_2=0.003, Φ_3=0.003, Φ_4=0.005, Φ_5=0.002,
0.0006	0.9420	0.9282	0.9150	0.9121	Φ_7=0.003, Φ_8=0.01 ψ_1=0.02,
0.0008	0.9404	0.9267	0.9134	0.9106	η_1=0.09, η_2=0.03, η_3=0.04, η_4=0.07, η_5=0.09,
0.0010	0.9388	0.9251	0.9119	0.9102	η_6=0.02, η_7=0.06, η_8=0.062, λ_1=0.03.

According to Table (3), a rise in the cleaning equipment's rate of failure (ϕ_1) reduces its lifespan by roughly 3% to 2.8%, while the hot Steckel mill's failure rate (ϕ_6) affects long-term unavailability by roughly 0.48 to 0.34 percent. The percent examination of columns (1) and (2) shows that the grinding device failure rate has a greater impact on system accessibility than the hot Steckel and cleansing equipment rates for failure. As a result, we plan to examine how the grinding equipment's repair and preventative service rates affect the system's functionality.

Table 4: Impact of Preventative Maintenance and Repair Rates of Grinding Machines on System Availability

ψ_1	0.2	0.4	0.6	0.8	1.0
$A\varepsilon$	0.9436	0.9440	0.9501	0.9578	0.9586
η_8	0.60	0.62	0.64	0.66	0.68
$A\boldsymbol{\varepsilon}$	0.9423	0.9436	0.9479	0.9623	0.9651

Table (4) shows that when the sub-system polishing equipment's replacement and preventative upkeep prices rise, so does its accessibility.

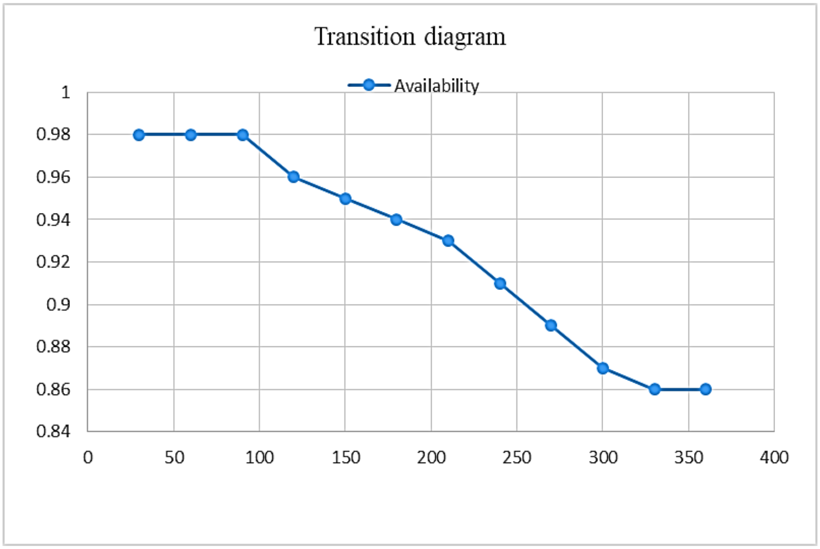

Figure 2*:* Analysis of Steel Industry Availability During Technician Inactivity

In contrast to conventional reliability models like the exponential and Weibull models (Al Rahbi; Hussain et al., 2024; Lai et al., 2023; Shabur et al., 2023; Xu et al., 2024), which often assume constant or predetermined failure and repair rates, the suggested methodology integrates subsystem-specific mechanical and operational characteristics. This facilitates more precise forecasting of system availability under fluctuating operational circumstances. Our findings indicate that including operational variability and maintenance schedules yields enhanced understanding of essential subsystems influencing overall dependability, underscoring the practical benefits of the suggested approach compared to traditional models. Figure 2 represents the analysis of steel industry availability during technician inactivity.

6. Conclusion

The milling equipment has a greater impact on the overall system's accessibility than other components, according to the study tables 1, 2, 3, 4 and figure 2. Thus, in terms of servicing, the hammering component is of the greatest significance and ought to be given the greatest attention.

The findings of this research provide engineers and plant managers with actionable insights that can be effectively implemented to enhance operational performance. The analyzed data can be utilized to guide targeted maintenance, improve equipment reliability, optimize process workflows, and increase overall system availability by identifying critical subsystems and operational bottlenecks. Furthermore, management can leverage these insights to prioritize resource allocation and implement data-driven strategies that enhance both performance and sustainability. This study offers a comprehensive examination of the mechanical and operational dynamics within steel production, emphasizing subsystem performance, process efficiency, and overall reliability. The results highlight that critical subsystems, such as the rolling mill and material handling units have a significant impact on total system availability and operational efficiency. By integrating quantitative and qualitative analyses, the research provides practical recommendations for optimizing maintenance schedules, improving equipment dependability, and refining production processes.

The technical and managerial implications of this study are clear: focused maintenance and monitoring of vital subsystems can reduce downtime, enhance process efficiency, and promote sustainable production practices. Additionally, plant management can use these findings to allocate resources more effectively and implement evidence-based operational strategies.

Future research will focus on validating the proposed reliability and operational behavior model through the collection of real-time data from steel plant operations. Incorporating live operational measurements will enable continuous improvement, model calibration, and more accurate forecasting of system reliability and efficiency under dynamic industrial conditions.

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