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Equalizing Leaf Spring Vehicle Suspension Synthesis Technologies Based on Modern Mathematical Simulation Modeling

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ABSTRACT

The research addresses the use of equalizing leaf spring vehicle suspension systems synthesis technology. The development of a vehicle with an equalising suspension system involves synthesising the required leaf spring load characteristics to achieve the desired vehicle smoothness, considering their potential implementation. This synthesis tasks must be solved using modern vehicle dynamics simulation modeling methods. The problem lies in the absence of a simplest dynamic equalizing leaf spring suspension model, which accounts for the leaf spring angular bending stiffness and can be used at the early stages of synthesis, when most of the leaf spring suspension parameters are still unknown. The research aims to develop a mathematical model for the equalizing leaf spring suspension dynamics in a multi-axle vehicle, considering the spring sheets angular bending stiffness. It should enable the use of this method in early-design vehicle motion models to synthesise the required suspension systems' load characteristics and subsequently analyse vehicle vibration load and force factors in various vehicle-operating modes. The developed spring dynamics model makes it possible to increase the accuracy of vehicle dynamics modeling by 3-6% and reduce the computer simulation time of vehicle dynamics in the range from 15 minutes to several hours. In practice, the developed mathematical model has found its application in the complex synthesis of equalising leaf spring suspension systems for multi-axle vehicle technologies. The research results indicate that the use of advanced few-leaf springs in equalising suspension systems for multi-axle cargo vehicles improves the vehicle smoothness by 5-13% compared to classic multi-leaf springs.

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1. Introduction

A vehicle leaf spring suspension system design always studies various vehicles' leaf springs parameters, properties, dimensions and structural features. Leaf springs are designed for different suspension strokes and loads. Collecting and analyzing the required information for newly designed vehicles is complicated. Even early research stages require information about the static and dynamic characteristics of the vehicle suspension system being developed. A completely analytical synthesis of leaf spring characteristics is impossible. The differential equations of vehicle suspension dynamics (Ozcan et al., 2023) are very complex and cannot have explicit, unambiguous solutions even when simplifying calculation schemes are used. Therefore, the synthesis problems of leaf spring suspension characteristics are solved using mathematical modelling methods, particularly through computer simulation dynamics models (Maljković et al., 2018). Vehicle movement simulation in various modes using various support surface types is the primary research method in modern vehicle springing theory. Modeling the vehicle movement along uneven tracks solves two critical tasks associated with the perspective vehicles design. The first task is to analyse the vehicle vibration load (Zou et al., 2023). The second task is to explore the loads in the suspension system used to calculate further the suspension components' strength and durability (Kong et al., 2019). The problem of interpreting the suspension system's load to determine the strength of its components is solved with a cost- and time-efficient research method, i.e., the virtual bench tests technology (Maksimov & Chichekin, 2021; Teli et al., 2019; Yang et al., 2022). This method allows for the quick and easy creation of virtual benches for the suspension system aggregates and simulation tests in the specified typical loading modes (Putra & Ikbal, 2021; Szczypinski-Sala et al., 2023; Teli & Deshmukh, 2021). However, this method is not applicable in the vehicle vibration loading studies. In vehicle vibration load research for the suspension system synthesising process, it is necessary to use a complete vehicle dynamics modelling method, considering the characteristics of its systems' reciprocal elastic interactions (Balike et al., 2010). (Hoyle, 2004; Kat, 2012; Toso et al., 2015)presents an example of elastic interaction forces using a dynamic leaf spring model, which can be integrated into a complete vehicle dynamics model. The currently published vehicle dynamics models mostly describe a general case of curved movement along various support surface types with a detailed description of the wheel-road interaction processes (Ajmi et al., 2017; Belrzaeg et al., 2021; Grigore et al., 2021; Isermann, 2021; Liu et al., 2024; PACEJKA, 2006) One of the most common schemes for creating a multi-axle vehicle suspension system is an equalizing suspension of two axles, where a leaf spring acts as an equalizing link (Fig. 1).

Figure 1: Equalizing Leaf Spring Vehicle Suspension

Known approaches to constructing a mechanical equalizing suspension dynamics mathematical model (Zhang et al., 2018) do not account for the leaf spring angular bending stiffness. The leaf spring is represented as a solid beam on an elastic base, which results in a decrease in dynamic modelling accuracy by 10-20% (Rubanov et al., 2023). There are known methods for calculating equivalent values of leaf spring vertical stiffness (Zhang et al., 2023), which simplify the dynamic models calculating processes, but do not fully reflect the physics of the processes occurring in the equalizing suspension leaf springs during bending. Another approach to equalising leaf spring suspension dynamics modelling is to represent a leaf spring package in the form of a finite element model (Kadziela et al., 2014; Mehta et al., 2019). However, this principle is applicable only when the leaf spring sheet package design is already known and verified, and is not appropriate at the early stages of suspension synthesis. There is another more accessible way of modeling the leaf springs dynamics. Multilink leaf spring dynamics models, which include a quasi-static solution taking into account the constraints for the leaf spring elasticity formation (Rill et al., 2022; Wang & Zhang, 2023). Such models are practical, relatively compact, and have sufficient accuracy across the entire range of applications. However, generating this type of leaf spring dynamics model requires extensive preliminary preparation, including calculating bending stiffness parameters between each model link. Hence, a clear gap in the research exists, specifically the absence of a simple vehicle equalising leaf spring suspension dynamics model that could be utilised at the early stages of synthesising multi-axle vehicle suspension characteristics with dynamic simulation modelling methods. The novelty of the vehicle equalizing spring suspension dynamics model proposed in this research lies in its simplicity, quickness of implementation and applicability at the early stages of the vehicle suspension design with most of the design parameters unknown. The leaf springs angular bending stiffness value can be calculated mathematically or experimentally, provided the leaf springs samples are available. The calculation time for the vehicle motion model dynamics is reduced by several times using the proposed equalizing leaf spring suspension model. Thus, the research aims to develop a mathematical model of the equalizing leaf spring suspension dynamics for a multiaxial vehicle, accounting for the leaf spring angular bending stiffness, which can be used in vehicle motion models at the early design stages to synthesize the required suspension systems load characteristics and to subsequently analyse the vehicle vibration load and the force factors arising in different vehicle suspension operation modes.

2. Materials and Methods

2.1 Modern Approaches to the Vehicle Dynamics Description

2.1.1 Dynamic Modeling Assumptions and Boundary Conditions

Considering the tasks of vehicle equalizing leaf spring suspension characteristics, synthesizing, which are to be solved by the dynamic simulation methods, the following requirements for the model describing the vehicle movement along the road supporting surface should usually be identified:

- The model should reflect the collaborative dynamics of the vehicle body and chassis, i.e. be a comprehensive model.
- The vehicle dynamics model must consider the non-holding nature of the connections imposed on the vehicle.
- The dynamics modeling results should be force (forces and moments) and kinematic (movements, speeds, accelerations, rotation angles) parameters of the vehicle movement, namely its body and chassis.
- The vehicle dynamics model should be parameterized as universally as possible in relation to a vehicle with various design parameters, as well as about the road conditions.
- The vehicle dynamics model implementation requires the use of the most efficient computational methods and software packages, such as MATLAB.Simulink, Simcenter AMESim, etc.

In most cases, the vehicle motion models are developed based on the following assumptions:

- The road profile is non-deformable and piecewise linear.
- The vehicle model is symmetrical with respect to the longitudinal plane passing through the vehicle body mass center.
- The tire-road surface point contact is assumed.
- The vehicle dynamic system *body – chassis – road* is generally non-linear.
- In specific driving modes, it is not uncommon for one or more wheels to lose contact with the road support surface, i.e., the wheels break off.
- When developing a vehicle motion model at the early stages of dynamics research, the strength aspects of its component system elements are not considered.

It is assumed that the model elements do not lose their functionality during the virtual dynamic experiment and operate within acceptable stresses. The latter assumption is logical, since at the early synthesis stages the object ignorance area is very significant. The strength aspects should be taken into account at the following stages of the object's structure development, based on the results obtained during the research on the object's dynamic parameters synthesis and evaluation.

2.1.2 Vehicle Body Spatial Movement Description

The vehicle body movement is a general case of *solid body movement* with a moving coordinate system $O_2X_2Y_2Z_2$ relative to the road stationary coordinate system OXYZ, as shown in Fig. 2.

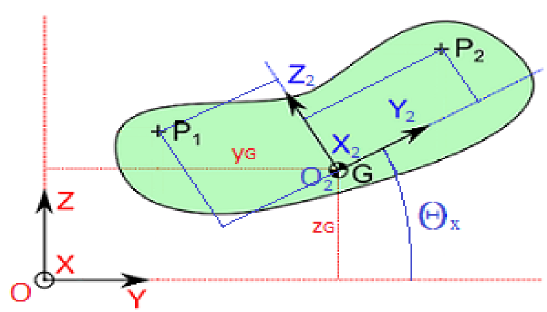

Figure 2: Spatial Location and Orientation of the Vehicle Body Solid Body Motion Model Relative to the Road Stationary Coordinate System

Indeed, there are no geometrical restrictions imposed on the vehicle body, and the instantaneous position can be determined by solving the differential equations of the vehicle body dynamics in vector form. In the general movement case, the solid body has six degrees of freedom (6-DOF). Each solid body dynamics in the movement model is described by Newton's and Euler's laws, as shown in equations (1) for the linear and (2) for the rotational DOF:

$$M_G \cdot a(G) = F(G) \tag{1}$$

$$J_G \cdot \frac{d\Omega_G}{dt} = T(G) \tag{2}$$

where M_G Is the solid body mass and the symbol G Indicates the point of the solid body mass center in every equation. J_G The inertia moment vector is relative to the principal inertial axes, i.e., X2, Y2, Z2, respectively. $a(G) = \frac{d\,\vartheta(G)}{dt} = \frac{d\,[x_G, y_G, z_G\,]}{dt}$ is the solid body mass center acceleration vector, where $\vartheta(G)$ is the solid body mass center speed vector and x_G, y_G, z_G Are the solid body mass centre position's longitudinal, lateral, and vertical coordinates, respectively, relative to the stationary coordinate system axes X, Y, and Z, respectively, in movement modelling time? t. $\Omega_G = \frac{d\,[\Theta_x, \Psi_y, \Phi_z\,]}{dt}$ is the angular velocity vector of the solid body moving coordinate system $O_2X_2Y_2Z_2$ relative to the road stationary coordinate system OXYZ and $\Theta_x,\ \Psi_y, \Phi_z$ are precession, nutation and self-rotation Euler angles respectively. $F(G)$ is the vector of forces applied to the solid body mass center and calculated using equation (3) and $T(G)$ is the vector of moments applied to the solid body mass center and calculated using equation (4):

$$F(G) = \sum_{j=1}^{n} F_{P_j} \tag{3}$$

$$T(G) = \sum_{j=1}^{n} (T_{P_j} + GP_j \times F_{P_j}) \tag{4}$$

Where F_{P_j} is the force, vector applied to any solid object point P_j ($j = 1 \dots n$) and n It is the quantity of force applied to solid body points (from forces that occur in the vehicle suspension systems). T_{P_j} is the vector of moments applied to any solid body point? P_j ($j = 1 \dots n$) and GP_j is the radius vector of the point P_j ($j = 1 \dots n$) location relative to the G solid body mass center.

2.1.3 An Example of a Vehicle Dynamic Model with a Dependent Rear Axle's Suspension System and Equalizing Axle's Links

When considering the mathematical motion model of a vehicle with a dependent suspension using the triple-axle vehicle example, where the rear two axles are connected by an on-board mechanical spring equalizer, for example, a leaf spring (Fig. 3), the vehicle body dynamics differential equations are described by Eq (1) and Eq (2). The vehicle wheels model with the tires and the road surface interaction as part of the vehicle movement model can be formed based on the materials presented in (Dugoff et al., 1970; Pacejka, 2000; Pacejka & Besselink, 1997).

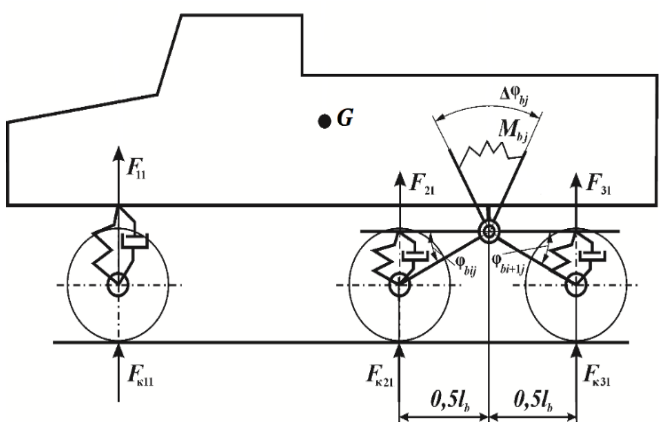

Figure 3: The Design Diagram of a Triple-axle Truck with an Equalizing Rear Dependent Suspension System

The forces in the vehicle suspension system F_{ij} (where $i = 1 \dots 3$ is the vehicle axle number and $j = 1 \dots 2$ is the vehicle side number) depend on the suspension relative deflection h_{ij} and deflection rate $\frac{dh_{ij}}{dt}$:

$$F_{ij} = F_{SPRING_{ij}}(h_{ij}) + F_{DAMP_{ij}}\left(\frac{dh_{ij}}{dt}\right) \tag{5}$$

where $F_{SPRING_{ij}}(h_{ij})$ is the force in the spring element of the JST vehicle side and $F_{DAMP_{ij}}\left(\frac{dh_{ij}}{dt}\right)$ This force is applied in the IST damping element of the JST vehicle side. The front axle dynamics (Fig. 4) are described by differential equations of motion for vertical oscillations along the Z_{most} axis and for angular oscillations relative to the X_{most} longitudinal axis passing through the axle mass center:

$$\begin{aligned} M_{most} \cdot \frac{d^2 z_{most}}{dt^2} &= F_{K_{ij}} + F_{K_{i(j+1)}} - F_{ij} - F_{i(j+1)} - M_{most} \cdot g \\ J_{most} \cdot \frac{d^2 \Psi_{most}}{dt^2} &= F_{K_{ij}} \cdot \frac{B_k}{2} - F_{K_{i(j+1)}} \cdot \frac{B_k}{2} - F_{ij} \cdot \frac{B_1}{2} + F_{i(j+1)} \cdot \frac{B_1}{2} \end{aligned} \tag{6}$$

where M_{most} is the vehicle axle mass, z_{most} is the vertical coordinate of the axle mass center relative to the road supporting surface, $F_{K_{ij}}$ and $F_{K_{i(j+1)}}$ are the vertical reactions in the contact point of *i^st^* wheel of *j^st^* and *(j+1)^st^* vehicle sides with the road supporting surface respectively, g is the gravity acceleration, J_{most} is the vehicle axle inertial moment relative to the X_{most} longitudinal axis, Ψ_{most} is the rotation angle of the vehicle axle relative to the X_{most} longitudinal axle, B_k is the vehicle axle wheel track, B_1 is the vehicle axle spring track.

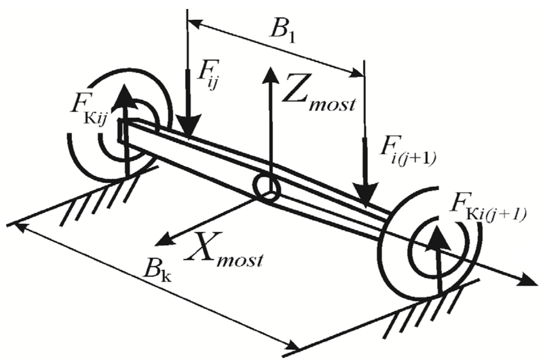

Figure 4: The Design Diagram for the Wheel Vehicle Axle Oscillations

2.2 A Mathematical Model of the Vehicle Equalizing Leaf Spring Suspension Dynamic, Taking into account the Leaf Spring Angular Bending Stiffness

The movement features of the vehicle rear axles with equalizing links in the rear trolley (Fig. 3) are concluded by analogy with the equations (6) in differential equations:

$$\begin{aligned} M_{most} \cdot \frac{d^2 z_{most}}{dt^2} &= F_{K_{ij}} + F_{K_{i(j+1)}} - F_{ij} - F_{i(j+1)} - \Delta F_{bj} - \Delta F_{b(j+1)} - M_{most} \cdot g \\ J_{most} \cdot \frac{d^2 \Psi_{most}}{dt^2} &= F_{K_{ij}} \cdot \frac{B_k}{2} - F_{K_{i(j+1)}} \cdot \frac{B_k}{2} - F_{ij} \cdot \frac{B_1}{2} + F_{i(j+1)} \cdot \frac{B_1}{2} - \Delta F_{bj} \cdot \frac{B_1}{2} + \Delta F_{b(j+1)} \cdot \frac{B_1}{2} \end{aligned} \tag{7}$$

where ΔF_{bj} and $\Delta F_{b(j+1)}$ are spring forces applied to the vehicle axle from the equalizer of *j^st^* and *(j+1)^st^* vehicle sides respectively due to leaf springs bending during h_{ij} rear axle suspension deflections.

The values of these spring forces, which arise because of the equalizer leaf spring sheets bending, can be calculated as:

$$\begin{aligned} \Delta F_{bj} &= \frac{M_{b_j}(\Delta\varphi_{bj})}{0{,}5 \cdot l_b} \\ M_{b_j}(\Delta\varphi_{bj}) &= C_M \cdot \Delta\varphi_{bj} \\ \Delta\varphi_{bj} &= \varphi_{bij} + \varphi_{b(i+1)j} \end{aligned} \tag{8}$$

where M_{b_j} is the equalizer spring moment, l_b is the equalizer leaf spring length, $\Delta\varphi_{bj}$ is the equalizer leaf spring rotation angle, C_M is the equalizer leaf spring angular bending stiffness. The values of the angular bending stiffness, denoted as C sub cap M, of the equaliser leaf springs are determined by synthesising the characteristics of the vehicle equalising leaf spring suspension system through simulation methods. The required range of values for the leaf springs angular bending stiffness parameter C_M can be estimated using the known techniques for the various configurations leaf springs stiffness calculating (Kadziela et al., 2014; Mehta et al., 2019; Rill et al., 2022; Rubanov et al., 2024; Wang & Zhang, 2023).

3. Results and Discussion

3.1 Research into the Dynamic of a Multi-axle Vehicle with a Rear Axles Equalizing Suspension System

3.1.1 Operability Verification for the Presented Mathematical Model of the Equalizing Leaf Spring Suspension Dynamics with the Vehicle Dynamics Simulation Methods

As part of this research, the developed mathematical model of equalizing the rear axle's suspension system dynamics, which considers the leaf spring angular bending stiffness, was integrated into a complete verified movement model of a triple-axle truck made in Simcenter AMESim Multi-Body Dynamics (Fig. 5). To test the operability of the developed mathematical model of the equalizing suspension system, considering the leaf spring angular bending stiffness, a simulation of the truck movement on a broken dirt road at 15 km/h was carried out. The simulation detected significant angles of leaf springs torsion (elastic bending) of the vehicle rear equalizing suspension in the range of ±5 deg.

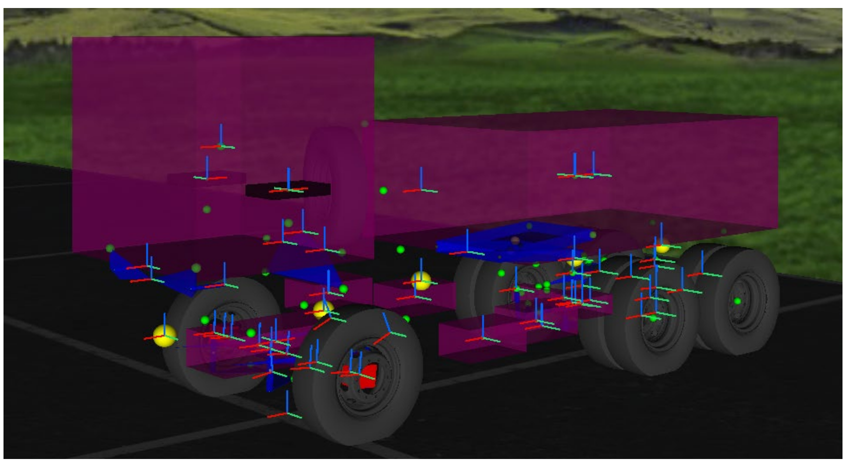

Figure 5: The Visualization of the Triple-axle Vehicle Dynamic Model in the Multi-Body Dynamic System

It is shown that when the vehicle body mass center vertical coordinate changes in movement relative to the dirt road support surface level (Fig. 6), the dependence of the change in the left side rear suspension equalizer angle $\Delta\varphi_{b1}$ The on-time form is shown in Fig. 7. These parameters indicate the operability of the developed mathematical model for equalising spring suspension dynamics and its approximation to real-world operation results.

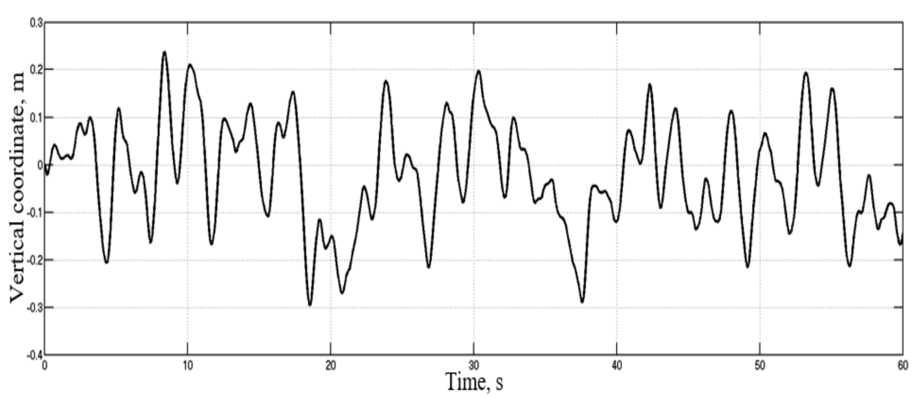

Figure 6: The Vehicle Body Mass Center Vertical Coordinate Changes in Simulation Time

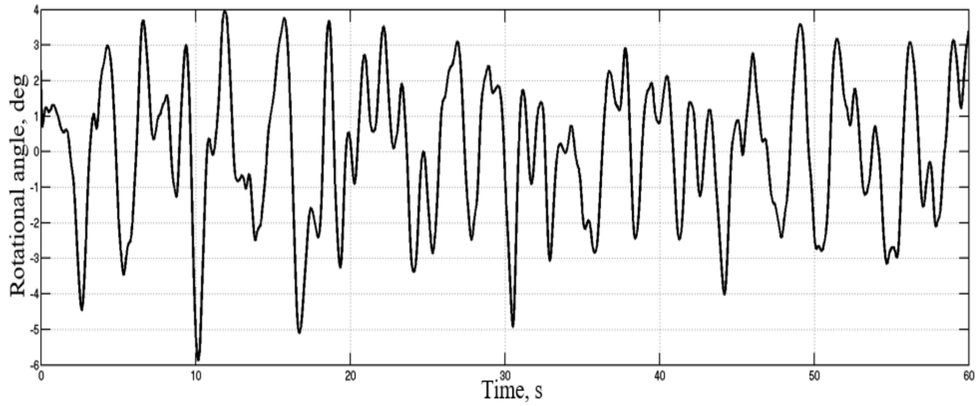

Figure 7: The Vehicle Left-side Rear Suspension Equalizer Torsion Angle $\Delta\varphi_{b1}$ Changes in Simulation Time

3.1.2 Validation of the Presented Mathematical Model of the Equalizing Leaf Spring Suspension Dynamics with the Vehicle Dynamics Simulation Methods

The triple-axle vehicle movement model described hereinabove, made in the Simcenter AMESim (Fig. 5) was verified with full-scale tests of a triple-axle dump truck (Fig. 8) to research the smoothness indicators. Then, comparison methods with experimental data recorded with measuring vibration accelerations special equipment (Fig. 8) showed an 8-12% error in modeling the fundamental configuration vehicle dynamics, where in the vehicle movement model the model of the rear axles equalizing suspension leaf spring was represented as a solid beam on an elastic base, according to the smooth-running criteria.

Figure 8: The Experimental Researches of the Triple-axle Dump Truck Smooth Running Characteristics

As part of this research, the modelling results of the vehicle movement with an integrated model of the equalising leaf spring suspension dynamics, presented in Eq. (8), were compared to the same triple-axle dump truck field tests (Fig. 8) results. The vehicle dynamics modeling error was 5-6%. Thus, it was found that the application of the developed mathematical model of the vehicle equalizing leaf spring suspension dynamics, which takes into account the leaf spring angular bending stiffness, makes it possible to increase the vehicle dynamics modeling accuracy by 3-6%. In addition, the computer time spent calculating vehicle motion models were compared using different methods to model the equalizing leaf spring suspension dynamics under identical conditions. The results showed that when using the classical approach (Zhang et al., 2018), where the leaf spring is represented as a solid beam on an elastic base, the computer simulation time for driving on a dirt road at 15 km/h for 60 seconds simulation time was 1 hour and 5 minutes. When applying the proposed mathematical model of the vehicle equalizing leaf spring suspension dynamics, which takes into account the leaf spring angular bending stiffness, the computer simulation time for a similar mode was 48 minutes. When using approaches with the leaf springs multi-link dynamics model's introduction (Rill et al., 2022; Wang & Zhang, 2023), the simulation time for a similar mode was 1 hour and 50 minutes. Furthermore, the use of approaches that feature the representation of spring sheets as finite element models (Kadziela et al., 2014; Mehta et al., 2019) increases the computer time costs for calculating a similar vehicle movement mode by more than 5 hours. Thus, it has been determined that the application of the developed mathematical model of the vehicle equalizing leaf spring suspension dynamics, considering the leaf spring angular bending stiffness, reduces the vehicle dynamics computer simulation time from 15 minutes to several hours.

3.2 Practical Application of the Developed Mathematical Model of the Equalizing Leaf Spring Suspension Dynamics in the Multi-axle Truck Suspension System Synthesis Processes

As part of this research, using the developed mathematical model of the equalising leaf spring suspension dynamics as part of the triple-axle vehicle movement model, the problem of synthesising rear axles equalising leaf springs suspension characteristics was solved. The aim was to determine the spring's stiffness required for the dump truck to run smoothly and to ensure the spring's realizability in terms of fulfilling the strength conditions. At the early research stages, the values of the leaf spring stiffness parameter were synthesised according to the vehicle smoothness criterion by dynamics simulation modelling. Further, in the process of synthesis and analysis of the obtained values of the vehicle's smooth running performance for each leaf spring configuration under consideration, procedures for forming geometric shapes and sheet parameters were performed based on the methodology from reference (Rubanov et al., 2024). Some of the most relevant spring configurations obtained from the synthesis have passed the stages of design and production of prototype samples. The manufactured leaf springs prototype samples were tested on a full-scale test bench equipment (Fig. 9) to experimentally confirm the stiffness characteristics obtained as a result of the synthesis at the early research stages.

Figure 9: The Vehicle Leaf Springs Bench Tests

The listed step-by-step order of actions collectively represents the technologies for synthesising vehicle equalising leaf spring suspension systems using modern mathematical modelling methods. Table 1 presents the synthesis results for the required characteristics of advanced few-leaf springs and classic multi-leaf springs, utilising the presented technology. It ensures equality in the basic total and connecting dimensions of the leaf springs, as well as the loads applied to them.

Table 1: The Results of Leaf Spring Parameter Synthesis for the Triple-axle Truck Equalizing Suspension System

Number of Spring Leaves	Stiffness	Spring Main Leave Strains According to the Von-Mises Criterion	Leaf Friction Presence
		For Advanced Few-Leaf Springs	
3	360 kN/m	1750…2050 MPa	Minimum
5	320 kN/m	1600…1850 MPa	Significant
		For Conventional Multiple-Leaf Springs	
9	2250 kN/m	900…1300 MPa	High
12	3120 kN/m	800…1000 MPa	Extremely High

According to the research results, which utilise vehicle dynamics simulation methods to synthesise spring parameters and characteristics, the use of advanced few-leaf springs in equalising suspension systems for multi-axle vehicles improves vehicle smooth running by 5-13% compared to classic multi-leaf springs.

4. Conclusions

A mathematical model of the equalizing leaf spring suspension dynamics, taking into account the leaf springs angular bending stiffness, has been developed. It can be integrated into vehicle dynamics simulation models. Simulation methods have confirmed the operability of the developed mathematical model. Its use allows for an increased accuracy in vehicle dynamics modelling by 3-6%. The developed mathematical model of the equalizing leaf spring suspension dynamics, which considers the leaf spring angular bending stiffness, reduces the vehicle dynamics computer simulation time from 15 minutes to several hours. In practice, the developed mathematical model has found its application in the technologies of complex synthesis of multi-axle vehicles equalizing leaf spring suspensions. According to the research results, the use of advanced few-leaf springs in multi-axle vehicles with equalising suspension systems, compared to classic multi-leaf springs, improves vehicle smoothness by 5-13%. In the future, the research will utilise the developed mathematical model of the equalising leaf spring suspension dynamics to synthesise the suspension system characteristics required for a specific vehicle with a given set of design and operational parameters. The advantage of the developed mathematical model is its implementation requires a minimum number of known initial leaf spring parameters. This fact allows it to be used in the early stages of vehicle development. In addition, the developed approach ensures independence from the leaf springs' structural design, including tolerances in leaf spring manufacture, making the developed model applicable at the earliest stages of suspension systems synthesis technologies.

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