Reliability analysis of turbine unit using Intuitionistic Fuzzy Lambda-Tau approach

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Article Info	ABSTRACT
Article history:	The current work presents a two phase Intuitionistic Fuzzy (IF) based
Received January 4, 2023 Revised March 30, 2023 Accepted April 17, 2023	framework, for investigating the reliability analysis of a Turbine Unit (TU) in a sugar mill process industry. Intuitionistic Fuzzy Lambda Tau (IFLT) approach-based series-parallel expressions have been applied for computing various reliability indices. For series arrangement OR gate transitions avpression has been used and for parallel arrangement AND gate
Keywords:	expression has been used, and for parameter analignment AND gate expressions has been used for calculation of reliability parameters for membership and non-membership function. Ear membership function
IFLT, Reliability parameters, Availability, Maintenance schedule, Reliability indices.	system's availability decreases by 0.000002% for spread value $\pm 15\%$ to $\pm 30\%$, further decreases by 0.000005% for spread value $\pm 30\%$ to $\pm 45\%$. While, non- membership function-based system's availability decreases by 0.000003% for spread value $\pm 15\%$ to $\pm 30\%$ and further decreases by 0.000007% for spread value $\pm 30\%$ to $\pm 45\%$. The reliability trends at various spreads lay the foundation of studying the failure behaviour of the TU and to plan a maintenance schedule.
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1. Introduction

The primary industry in rural India is the sugar industry. One of the foundational sectors of the Indian economy is the sugar mill industry, which accounts for almost 1% of the nation's GDP (Solomon, 2011a). The sugar industry in India provides a living for many farmers and their families. It employs over 5 lakh people both trained as well as semi-skilled workers directly involvedin sugar mills and allied companies around the nation. With an impact on the rural livelihoods of more than 50 million sugarcane farmers, the sugar industry is a substantial agro-based sector (Solomon, 2011b). In a sugar mill, Turbine Unit (TU) is the pivotal unit because it supplies steam which is required for processing of sugar. Processing of sugar include, juice heating drying, evaporation and crystallization of sugar etc. Apart from processing, the superheated steam is utilised in TU for generation of electricity. TU is the most critical and complex system in a sugar mill industry because it fulfils the requirement of steam and electricity. Due to complexity of the considered system, if any equipment deviates its intended function, it results in fluctuation of power and sometimes tripping of the system. To avoid such conditions, failure free operation of a complex TU is of paramountcy.

For complex industrial systems, failure prediction with high accuracy is a difficult undertaking due to the unavailability of raw data or information(Gopal & Panchal, 2023a). Moreover, raw data available from various sources contains high degree of uncertainties/vagueness led to inaccuracy in results and poor maintenance schedule. It's not that a unit stops working as a result of inadequate maintenance practises, but poor maintenance practises result in operational mishaps, loss of production, poor-quality products, and even the shutdown of the entire mill. Intuitionistic Fuzzy Set (IFS) have been proved as highly efficient to consider the degee of indeterminacy in collected data. Therefore, in order to encounter these issues' reliability analysis of TU has been presented in this work.

2. Literature Background

To study the performance issues of various industries, such as paper mills, thermal power plants, milk process industries, and fertiliser industries, many researchers have so far proposed various frameworks. Markovian chain analysis-based Chapman-Kolmogorov differential equations to compute several reliability indices and analyse failure behaviour was one of the earlier used models. Kumar et al., (1992)applied differential equations based on Markov approach for studying stochastic failure behaviour on the basis of availability of the unit. Arora & Kumar, (1997) proposed Markov model based on mathematical equations for computing availability of ash treatment unit. Lisnianski et al., (2012) presented the application of Markov methodology to conduct the reliability analysis of power producingunit. Sharma & Vishwakarma, (2014)applied the Markov technique to investigate the feeding system's performance analysis. In a fertiliser production unit, performance analysis was carried out by applying Markov birth-death equations to study failure of the unit (Aggarwal et al., 2015). Again, reliability estimation of the crystallisation unit in the sugar mill was carried out using Markov mathematical modelling equations (Aggarwal et al., 2017). In order to analyse the failure dynamics of the evaporation system in the sugar millMarkov technique was employed to compute the reliability parametersin a chemically treated sugar industry(Saini & Kumar, 2019).

The drawback of Markovian model lies in the fact that it was based on crisp set theory-based data/information and did not take into consider uncertainty/vagueness features in the gathered data, and hence an element of uncertainty always continues to affect the accuracy of results. The advent of Zadeh's fuzzy set theory (Zadeh, 1965) has overcome the demerit of Markovian chain model. Fuzzy set theory was applied to consider the uncertainties/vagueness in the data to study perfmance issues of industrial system.

To tabulate performance parameters like Reliability, Availability and Maintainability (RAM), Knezevic & Odoom, (2001), proposed FLT technique. Apictorial representation, Petri Net (PN) model was also incorporated to model the series-parallel configuration of industrial units. In order to assess and analysethe performance issues of washing unit in the paper industry, a structured framework based on fuzzy set theory was applied (Sharma et al., 2008). Komal et al., (2010) proposed a Genetic Algorithms based Fuzzy Lambda-Tau (GABLT) techniquefor computing RAM parameters, and demonstrated the application of framework to maximise press and washing subsystem's availability in the paper mill industry. Various other researchers also applied FLT approach for analysing the performance aspects, on the basis of reliability parameters of process industries- Robotic system (Sharma et al., 2012a); Paper mill (Sharma et al., 2012b); Power generating unit (Panchal & Kumar, 2016a); Water treatment plant (Panchal & Kumar, 2016b); Chlorine plant (Panchal et al., 2019); Chemical industry (Gopal et al., 2021); Dairy industry(Gopal & Panchal, 2023c). Moreover IFS find wider application with Multi Criteria Decision Making Approach (MCDM) approach also.Researchers have applied IFS based MCDM approaches in various fields- Healthcare services (Ecer & Pamucar, 2021);Logistics centre selection (Pamučar & Ćirović, 2015); Bidder selection (Pamučar et al., 2017);Tribological process (Kumar et al., 2022);Firing positions' locations selection(Jokić et al., 2021);Barrier selection in supply chain management (Biswas & Das, 2020).

In the above listed work, fuzzy theory concept has been used to encounter the uncertainties/ vagueness present in the data. The hesitation element in Degree of Membership (DOM) was not present in these fuzzy based models, in other words, indeterministic concept was not considered while carrying the performance analysis of industrial systems, consequently, problems has been encountered regarding reliability-based results. This drawback has overcome by implementing IFS theory which is capable to mathematically map the hesitation element in the expert's knowledge, gained importance in evaluating the performance evaluation of industrial systems with high accuracy and many researchers have applied this concept in different areas i.e., Garg, (2014) presented the application of Intuitionistic Fuzzy Lambda- Tau(IFLT) approach for analysing the failure performance of pulping unit. Yadav et al., (2014)presented the application of vague Lambda Tau approach to carry reliability analysis of ObjectOriented System (OOS). Vishwakarma & Sharma, (2016) applied vague Lambda Tau in a manufacturing unit to study and analyse its failure behavioural analysis. Fault tree analysis analysis was proposed to analyse failure dynamics of oil tank unit (Kumar & Kaushik, 2020). Kushwaha et al., (2021)applied IFLTapproach for studying the failure dynamics of cutting system based on reliability parameters. Again, Kushwaha et al., (2022) applied IFLT approach in a chemically treated clarification unit of a sugar plant.

It has been found from the literature review that the intuitionistic fuzzy modeling-based reliability analysis utilising IFLT approach of TU was not reported in the literature. To bridge the identified gap, IF modelling-based reliability analysis has therefore been proposed in work, and is presented with its application of turbine unit in a sugar mill industry located in western Uttar Pradesh, India.

3. Proposed framework

For the purpose of conducting the reliability analysis of TU in the sugar mill industry, a two-phase framework has been presented. The first stage of the framework begins with the gathering of maintenance data, which consists of quantitative data from maintenance expert's feedback that is pertinent to TU of sugar mill. In the second phase, a PN model was developed using information of the series-parallel arrangement of units. Different reliability parameters were calculated at different levels of uncertainty using the AND/OR gate reliability formulae for both the membership and non-membership function. In terms of the various trends of the tabulated reliability trends, the failure dynamics of the turbine unit system under consideration were studied. Figure. 1 depicts the two-phase flowchart of the proposed framework.



Figure 1. A two-phase flowchart of the proposed framework

4. Notions of IFS

4.1. Basic notions of IFS theory

Definition 1: An IFS in a universal set *S* is given by Eq. (1) as:

$$\widetilde{A} = \left\{ \langle x, \boldsymbol{\mu}_{\widetilde{A}}(x), \boldsymbol{\vartheta}_{\widetilde{A}}(x) \rangle, x \in S \right\}$$
(1)

Where, x is an element; $\mu_{\tilde{A}}(x)$ is a membership function; $\vartheta_{\tilde{A}}(x)$ is a non-membership function given by Eqs. (2) - (3) as:

$$\boldsymbol{\mu}_{\tilde{A}}: S \to [01], x \in S \to \boldsymbol{\mu}_{\tilde{A}}(x) \to [01]$$
⁽²⁾

$$\boldsymbol{\vartheta}_{\tilde{\boldsymbol{\lambda}}}: S \to [01], x \in S \to \boldsymbol{\vartheta}_{\tilde{\boldsymbol{\lambda}}}(x) \to [01] \tag{3}$$

$$\boldsymbol{\mu}_{\tilde{A}}(x) + \boldsymbol{\vartheta}_{\tilde{A}}(x) \le 1, x \in S \tag{4}$$

$$\boldsymbol{\pi}_{\tilde{A}}(x) = 1 - \boldsymbol{\mu}_{\tilde{A}}(x) + \boldsymbol{\vartheta}_{\tilde{A}}(x), x \in S$$
(5)

Where, $\pi_{\tilde{A}}(x)$ is degree of indeterminacy or hesitation. **Definition 2:** α cut of IFS is given by Eqs. (6) – (7) as:

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$$P^{(\boldsymbol{\alpha})} = \left\{ x \in S : \boldsymbol{\mu}_{\widetilde{A}}(x) \ge \boldsymbol{\alpha} \right\}$$
(6)

$$P^{(1-\boldsymbol{\alpha})} = \left\{ x \in S : 1 - \boldsymbol{\vartheta}_{\tilde{A}}(x) \ge \boldsymbol{\alpha} \right\} = \left\{ x \in S : \boldsymbol{\vartheta}_{\tilde{A}}(x) \le 1 - \boldsymbol{\alpha} \right\}$$
(7)

Where α is in the range of $0 \leq \alpha \leq 1$.

4.2. IFLT approach

FLT approach is a well-known efficient approach, developed for reliability analysis of industrial system arranged in series-parallel configuration (Gopal & Panchal, 2023b; Knezevic & Odoom, 2001). The results under the implementation of this approach are based on membership values (in closed interval 0 to 1) only for considering uncertainty/vagueness in the quantitative raw data available from various sources. In the past, this approach has been applied by many researchers in order to carry the reliability analysis of different industrial system under uncertainty (Gopal & Panchal, 2021; Panchal et al., 2019; Srivastava et al., 2020).Hesitation element in membership function values was not considered under traditional FLT approach, which was one of the major drawback responsible for low accuracy in results(Garg, 2014; Gopal & Panchal, 2022).To overcome this drawback, the introduction of IF concept in traditional FLT approach prove to be very useful for achieving highly accurate results(Garg, 2014). IFLT approach considers both membership and non- membership function-based values to consider the hesitation element, leads to high degree of accuracy in the reliability results. The various steps followed in IFLT approach are given as follows:

Step 1: Construct PN model of the turbine unit utilising AND/OR gate symbols (Figure 2).



Figure 2. PN model (a) OR gate (b) AND gate combination

Step 2: Collect the failure rate and repair time data for turbine unit Step 3: Convert the crisp failure and repair time data into fuzzified form using Eqs. (8) and (9).

$$\boldsymbol{\mu}_{\tilde{A}}(x) = \begin{cases} \boldsymbol{\mu}\left(\frac{x-a}{b-a}\right); a \leq x \leq b \\ \boldsymbol{\mu}; x = b \\ \boldsymbol{\mu}\left(\frac{c-x}{c-b}\right); b \leq x \leq c \\ 0; otherwise \end{cases}$$

$$1 - \boldsymbol{\vartheta}_{\tilde{A}}(x) = \begin{cases} 1 - \boldsymbol{\vartheta}\left(\frac{x-a}{b-a}\right); a \leq x \leq b \\ 1 - \boldsymbol{\vartheta}; x = b \\ \boldsymbol{\mu}\left(\frac{c-x}{c-b}\right); b \leq x \leq c \\ 0; otherwise \end{cases}$$

$$(8)$$

$$(9)$$

Step 4: Using AND/OR transition expression (Eqs. 10-17) compute membership and non-membership values for the top event.

Membership function expression AND Gate Transition Expression

$$\boldsymbol{\lambda}^{\boldsymbol{\alpha}_{\boldsymbol{\mu}}} = \begin{bmatrix} \prod_{i=1}^{n} \{ (\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i1} \} \sum_{j=1}^{n} \prod_{\substack{i=1\\i\neq j}}^{n} \{ (\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i1} \}, \\ \prod_{i=1}^{n} \{ -(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i3} \} \sum_{j=1}^{n} \prod_{\substack{i=1\\i\neq j}}^{n} \{ (\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i3} \} \end{bmatrix}$$
(10)

$$\boldsymbol{\tau}^{\boldsymbol{\alpha}_{\boldsymbol{\mu}}} = \left[\frac{\prod_{i=1}^{n} \{(\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i1}\}}{\sum_{j=1}^{n} \left[\prod_{i=1}^{n} \{-(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i3}\}\right]}, \frac{\prod_{i=1}^{n} \{(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i3}\}}{\sum_{j=1}^{n} \left[\prod_{i=1}^{n} \{(\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i1}\}\right]}\right]$$
(11)

OR Gate Transition Expression

$$\boldsymbol{\lambda}^{\boldsymbol{\alpha}_{\boldsymbol{\mu}}} = \left[\sum_{i=1}^{n} \{(\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i1}\}, \sum_{i=1}^{n} \{-(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i3}\}\right]$$
(12)

$$\boldsymbol{\tau}^{\boldsymbol{\alpha}_{\boldsymbol{\mu}}} = \begin{bmatrix} \frac{\sum_{i=1}^{n} \left[\{ (\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i1} \}, \{ (\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i1} \} \right]}{\sum_{j=1}^{n} \left[\{ - (\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \boldsymbol{\alpha} + \boldsymbol{\lambda}_{i3} \} \right]}, \\ \frac{\sum_{j=1}^{n} \left[\{ - (\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i3} \}, \{ - (\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\tau}_{i3} \} \right]}{\sum_{i=1}^{n} \left[\left\{ (\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\mu}}}{\boldsymbol{\mu}_{i}} + \boldsymbol{\lambda}_{i1} \} \right]} \right] \end{bmatrix}}$$
(13)

Non-membership functionexpression AND Gate Transition Expression

$$\boldsymbol{\lambda}^{(\boldsymbol{\alpha}_{i-\boldsymbol{\vartheta}})} = \begin{bmatrix} \prod_{i=1}^{n} \{ (\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i1} \} \sum_{j=1}^{n} \prod_{i=1 \ i \neq j}^{n} \{ (\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i1} \}, \\ \prod_{i=1}^{n} \{ -(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i3} \} \sum_{j=1}^{n} \prod_{i=1 \ i \neq j}^{n} \{ -(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i3} \} \end{bmatrix}$$

(14)

$$\boldsymbol{\tau}^{(\boldsymbol{\alpha}_{i-\boldsymbol{\vartheta}})} = \left[\frac{\prod_{i=1}^{n} \{(\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i1}\}}{\sum_{j=1}^{n} \left[\prod_{i=1}^{n} \{-(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i3}\}\right]}, \frac{\prod_{i=1}^{n} \{-(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i3}\}}{\sum_{j=1}^{n} \left[\prod_{i=1}^{n} \{(\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i1}\}\right]} \right]$$
(15)

OR Gate TransitionExpression

$$\boldsymbol{\lambda}^{(\boldsymbol{\alpha}_{i-\boldsymbol{\vartheta}})} = \left[\sum_{i=1}^{n} \{(\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i1}\}, \sum_{i=1}^{n} \{-(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i3}\}\right]$$
(16)

(17)

$$\boldsymbol{\tau}^{(\boldsymbol{\alpha}_{i},\boldsymbol{\vartheta})} = \begin{bmatrix} \frac{\sum\limits_{i=1}^{n} \left[\{(\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i1} \}.\{(\boldsymbol{\tau}_{i2} - \boldsymbol{\tau}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i1} \} \right]}{\sum\limits_{j=1}^{n} \left[\{-(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i3} \} \right]},\\ \frac{\sum\limits_{i=1}^{n} \left[\{-(\boldsymbol{\lambda}_{i3} - \boldsymbol{\lambda}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i3} \}.\{-(\boldsymbol{\tau}_{i3} - \boldsymbol{\tau}_{i2}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\tau}_{i3} \} \right]}{\sum\limits_{j=1}^{n} \left[\left\{ ((\boldsymbol{\lambda}_{i2} - \boldsymbol{\lambda}_{i1}) \frac{\boldsymbol{\alpha}_{\boldsymbol{\vartheta}}}{1 - \boldsymbol{\vartheta}_{i}} + \boldsymbol{\lambda}_{i1} \} \right\} \right]} \right]}$$

Step 4 Tabulate various reliability parameters at different α cut values using expressions

		D 11 1 111	•
Table	Ι.	Reliability	expressions
		1.001100011101	•

Reliability Indices	Expressions
MTTFs	$\frac{1}{\lambda_s}$
MTTRs	$\frac{1}{\mu_s}$
MTBF	MTTFs + MTTRs
Reliability (Rs)	$e^{-\lambda s.t}$
Availability (As)	$\frac{\mu_s}{\mu_s + \lambda_s} + \frac{\lambda_s}{\mu_s + \lambda_s} e^{-(\mu_s + \lambda_s)t}$

Step 5: Compute the defuzzied values (u^*) of reliability parameters using Eqn. (18).

$$u^{*} = \frac{\int_{u_{1}}^{u_{2}} \mu_{out}.(u) du}{\int_{u_{1}}^{u_{2}} \mu_{out}.(u) du}$$
(18)

5. Industrial case study

The case study considered for exemplifying the propsed framework is tubine unit. The steam generated in boiler is fed to the turbine. As the steam impinges the blade of turbine it starts rotating. The schematicarrangement representing the series-parallel arrangement of the considered TU in a sugar mill industry is shown in figure 3. The TU selected as a case study in the proposed work generates 28 MW and fulfil the requirement of steam for processing of sugar as well as demand of electricity is met by the generation. The brief description of TU and its various components are as follows:

(*i*)**Main oil pump**: It is one in number and connected in series configuration. It Supply lubricating oil to all the pumps.

(*ii*)Auxiliary oil Pump: Supply lubrication oil when turbine is required to stop during shutdown. It is one in number and connected in series configuration.

(*iii*) Emergency oil pump: When the turbine is in the turning gear, this is employed to give the lower flow that is necessary. It is one in number and connected in series configuration.

(iv)Auxiliary oil control pump: It supplies oil to hydraulic system and governor. It is one in number and connected in series configuration.

(v)Condensate extraction pump: It is employed to extract the exhaust steam and supply to the condenser.



Figure.3 A schematic diagram of TU

Using basic symbol of OR/ AND gate, PN model of TU was developed as per schematic diagram (figure 3) and shown in figure 4.



Figure.4 PN model of TU

TU- Turbine Unit, MT- Main oil tank, AP- Auxiliary oil pump, EP- Emergency oilpump, AC - Auxiliary control oil pump, OH- Oil header, TB- Turbine CP-Condensate extraction pump.

Quantitative information was gathered from a variety of sources, including the maintenance log book and the opinions of experts, about the failure rate and repair times of different subsystems and components of the turbine unit. The collected data is shown in Table 2.

Sr. No	Components	Failure Rate (h ⁻¹)	Repair Time(hrs)
1	Main oil tank (MT)	1.9841x 10 ⁻⁴	12
2	Auxiliary oil pump (AP)	1.9841x 10 ⁻⁴	4
3	Emergency oil pump (EP)	1.9841x 10 ⁻⁴	4
4	Auxiliary control oil pump (AC)	1.9841x 10 ⁻⁴	4
5	Oil header (OH)	1.9841x 10 ⁻⁴	12
6	Turbine (TB)	1.9841x 10 ⁻⁴	4
7	Condensate extraction pump (CP)	2.0833x 10 ⁻³	12

Table 2. Quantitative data under TU (Failure rate and repair time)

Triangular Intuitionistic Fuzzy Numbers (TIFN) have been computed at various spreads i.e., $\pm 15\%$, $\pm 30\%$ and $\pm 45\%$ applying equations (8) – (9) utilising the data (Table 2). The converted TIFN at $\pm 15\%$ spread for component Auxiliary control oil pump (AC) of TU is shown in figure 5.



Figure 5. TIFN \pm 15% spread value for MT

Similarly, TIFNs values for other components were also tabulated at other spread i.e. $\pm 30\%$ and $\pm 45\%$ spreads. Using TIFN values for all components in the equation (10) – (17) reliability parameters were tabulated at different α cut values (varies from 0-1) using various relations as shown in table 3 (a) and 3 (b).

Left spread					Right spread					
DOM	Failure	Repair	Reliability	MTBF	Availability	Failure	Repair	Reliability	MTBF	Availability
	rate	time				rate	time			
1	0.0033	5.0193	0.6376	373.5326	0.9927	0.0027	2.7491	0.5769	305.8183	0.9838
0.9	0.0032	4.6641	0.6297	363.4576	0.9919	0.0028	2.9715	0.5842	312.8947	0.9853
0.8	0.0031	4.3326	0.6219	353.9150	0.9910	0.0028	3.2089	0.5916	320.3116	0.9866
0.7	0.0031	4.0229	0.6142	344.8640	0.9901	0.0029	3.4624	0.5990	328.0937	0.9879
0.6	0.0030	3.7333	0.6065	336.2679	0.9890	0.0030	3.7333	0.6065	336.2679	0.9890
0.5	0.0029	3.4624	0.5990	328.0937	0.9879	0.0031	4.0229	0.6142	344.8640	0.9901
0.4	0.0028	3.2089	0.5916	320.3116	0.9866	0.0031	4.3326	0.6219	353.9150	0.9910
0.3	0.0028	2.9715	0.5842	312.8947	0.9853	0.0032	4.6641	0.6297	363.4576	0.9919
0.2	0.0027	2.7491	0.5769	305.8183	0.9838	0.0033	5.0193	0.6376	373.5326	0.9927
0.1	0.0026	2.5407	0.5698	299.0603	0.9822	0.0033	5.4000	0.6456	384.1852	0.9934
0	0.0025	2.3455	0.5627	292.6003	0.9805	0.0034	5.8086	0.6538	395.4663	0.9941

Table 3(a). Left and right spread values of various reliability parameters for membership function at $\pm 15\%$ spread.

Table 3(b). Left and right spread values of various reliability parameters for non-membership function \pm 15% spread.

Left spread				Right spread						
DOM	Failure rate	Repair time	Reliability	MTBF	Availability	Failure rate	Repair time	Reliability	MTBF	Availability
1	0.0031	4.2863	0.6206	352.3730	0.9909	0.0028	3.2476	0.5928	321.5828	0.9868
0.9	0.0031	4.0436	0.6145	345.3051	0.9901	0.0029	3.4470	0.5986	327.6957	0.9878
0.8	0.0030	3.8134	0.6086	338.5173	0.9893	0.0030	3.6570	0.6045	334.0483	0.9887
0.7	0.0029	3.5950	0.6026	331.9935	0.9884	0.0030	3.8782	0.6105	340.6546	0.9895
0.6	0.0029	3.3877	0.5968	325.7189	0.9875	0.0031	4.1115	0.6165	347.5300	0.9903
0.5	0.0028	3.1909	0.5909	319.6797	0.9865	0.0031	4.3574	0.6225	354.6910	0.9911
0.4	0.0028	3.0040	0.5852	313.8632	0.9855	0.0032	4.6169	0.6287	362.1556	0.9918
0.3	0.0027	2.8265	0.5795	308.2576	0.9844	0.0032	4.8907	0.6348	369.9431	0.9924
0.2	0.0026	2.6579	0.5738	302.8519	0.9832	0.0033	5.1799	0.6411	378.0750	0.9930
0.1	0.0026	2.4977	0.5682	297.6359	0.9819	0.0034	5.4855	0.6474	386.5742	0.9936
0	0.0025	2.3455	0.5627	292.6003	0.9805	0.0034	5.8086	0.6538	395.4663	0.9941

DOM: Degree of Membership

The calculated values TIFN are used in equations (10) - (17) and different reliability parameters (using table 1) for the top value of TU has been computed as per PN model (Fig 4) at various α cut values. Here as the turbine runs for 24 x7 = 168 hours of operation so,168 hours of mission time are taken into consideration asper discussion with plant manager. The values of various reliability parameters at 0 -1 α cut values (with increment of 0.1) for \pm 15% for left and right spread is given in Tables 3 (a) and 3 (b) respectively.

Likewise, reliability parameters at \pm 30 and \pm 45% spreads for different α cut values were also calculated but are not given due space constraints. Further, using equation (18), fuzzified output values for the various reliability parameters at various spreads (\pm 15%, \pm 30% and \pm 45%) are converted into crisp values and are shown in Table 4.

Reliability Parameters	Membership type	(15% Spread)	(30% Spread)	(45% Spread)
Failure Rate	Ι	0.003075	0.003175	0.003274
	II	0.003022	0.003067	0.003113
Repair Time	Ι	4.391131	5.713504	8.021230
	II	4.146807	5.112403	6.895750
MTTR	Ι	0.227732	0.175024	0.124669
	II	0.241149	0.195603	0.145017
MTBF	Ι	442.600644	485.963149	572.071504
	II	438.127703	473.510937	547.211674
Reliability	Ι	0.618034	0.632351	0.649528
·	II	0.612355	0.620570	0.631198
Unreliability	Ι	0.381966	0.367649	0.350472
	II	0.387645	0.379430	0.368802
Availability	Ι	0.999968	0.999966	0.999961
5	II	0.999967	0.999964	0.999957

Table 4. Reliability parameters at various spreads under TU

I – Membership function, II- Non- membership function

6. Failure dynamic analysis under TU

The failure rate increased by 0.032520% for spread values between 15% to 30% for the membership function and by 0.031181% for spread values between 30 to 45%. The non-membership function values showed similar trend. It is increased by 0.014891% for spread values 15% to 30%, and by 0.014990% for spread values between 30% and 45% as depicted from table 4.

Contrarily, repair time for both membership and non- membership function values increased. For membership function with spread value $\pm 15\%$ to $\pm 30\%$, repair time is increased by 0.301146% and for spread value ± 30 to $\pm 45\%$ magnitude of repair time was increased by 0.403907%. For case of non-membership, repair time was increased by 0.232853% for $\pm 15\%$ to $\pm 30\%$ spread, and further increased by 0.348828% for ± 30 to $\pm 45\%$ spread value.

Moreover, the MTBF of the considered TU showed an increasing trend. For spread values 15% - 30%, MTBF was increased by 0.097972%, for spread values 30% - 45% for membership function, MTBF was further increased by 0.177191%. As the spread increased from 15% to 30% for non-membership cases, it was increased by 0.080760%, and it was then further increased by 0.155647% when the spread increased from 30% to 45%. Also, there was an increasing trend depicted in reliability of the TU.

When the spread was increased from 15% to 30%, reliability of the considered unit increased by 0.023165%. It is then increased by 0.027164% when the spread was increased from 30% to 45% for the membership function. When the spread was increased from 15% to 30%, it was increased for non-membership cases by 0.017126%, and it was increased again by 0.017126% when the for spread value 30% - 45%. Furthermore, for both membership and non-membership functions, availability depicted decreasing trend. Availability is decreased by 0.000002% for membership functions for spread value 15% - 30% and by 0.000005% for membership functions with spread values between 30% and 45%. For case of non-membership, it was decreased by 0.000003% for \pm 15% to \pm 30% spread, and further decreased by 0.000007% for \pm 30 to \pm 45% spread value. The trends of reliability parameters have also been presented in the form of graphs in figure 5 (a) - (g).







(b)



Reliability analysis of turbine unit using Intuitionistic Fuzzy Lambda-Tau approach (Dilbagh Panchal)



(d)







(f)



(g)

Figure 5(a)-(g). Trends of reliability parameters

7. Conclusions

IFLT approach has been applied to compute various reliability parameters at different uncertainty level values within IFS set of [0.6, 0.8]. Three crucial reliability parameters such as failure rate and repair time and availability will be imperative to frame a maintenance schedule for the turbine unit. The former two reliability parameter shows increasing trends while availability shows decreasing trend at different level of spreads, as depicted from table 3. The proposed framework covers the hesitation element inevitable in the data obtained from the maintenance personnel form the industry. Also, the reliability parameters for both membership and non- membership function is more flexible in making maintenance decision unlike fuzzy FLT approach, which was based on membership function values only.

8. Managerial implications, limitations and future scope of work

The consideration of hesitation effect in the raw data obtained for the TU results in accurate computation of accurate reliability parameters. These reliability parameters form the basis of designing correct maintenance policy for the unit. Consequently, the frequent failure of the unit could be mitigated and long run availability of system is ensured. The reliability results calculation does not consider the interdependencies of the various components are the limitation of the work.

The current work presents the reliability analysis of the unit under IF environment. As this approach is based on synthesis point of view, means all the basic events (components) are combined to compute the reliability parameters of top event value. In future, the research direction likely to entails risk assessment under IF fuzzy approach. Moreover, the obtained ranking of results under risk assessment approach could be compared to well established IF -Multi Criteria Decision Making (MCDM) approach like IF-Technique for Order of Preference by Similarity to Ideal Solution (IF-TOPSIS), IF-COmplex PRoportional ASsessment (IF-COPRAS), IF-Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) approaches..

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